12MS Graphing Workbook

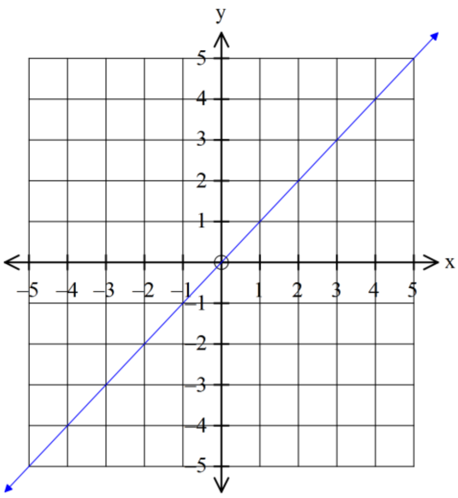
Absolute Value Graphs

The **absolute value** is the positive version of any number. It is expressed using two vertical lines around a number

e.g. The absolute value of 20 or |20| is 20. The absolute value of −20 or |−20| is also 20.

The graph $y = x$ can be drawn by calculating co-ordinates

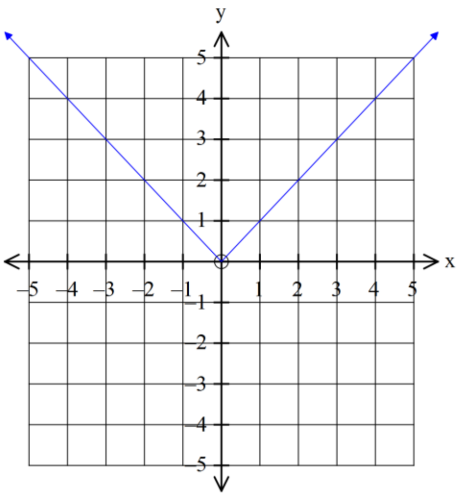
x	$y = x$
−3	−3
−2	−2
−1	−1
0	0
1	1
2	2
3	3



For an absolute value graph $y = |x|$, our table would look like this:

x	$y = \|x\|$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3

These numbers become positive as the absolute value of a negative number is its positive



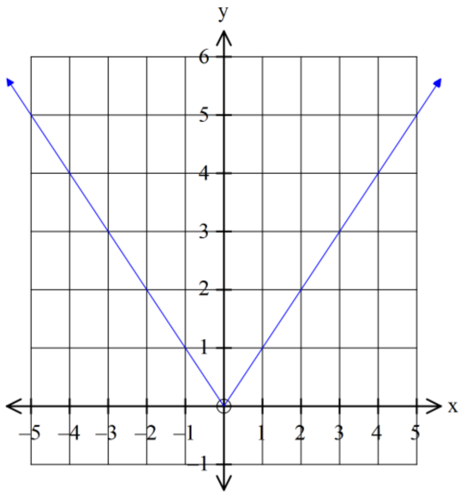
To get an absolute value graph, go into the GRAPH function.

Press OPTN and select F5 (NUM) then F1 (Abs).

Anything that is between the absolute value lines must be put in brackets after the function “Abs”

The standard absolute value graph is $y = |x|$. It has a vertex (or minimum point) at (0, 0)

$y = |x|$ can be transformed in the following ways:



- $y = |x| + 2$ shifts $y = |x|$ two units up
- $y = |x| - 2$ shifts $y = |x|$ two units down
- $y = |x + 2|$ shifts $y = |x|$ two units left
- $y = |x - 2|$ shifts $y = |x|$ two units right
- $y = 2|x|$ makes $y = |x|$ skinnier
- $y = \frac{1}{2}|x|$ makes $y = |x|$ fatter

If we combine all these transformations, a **general equation** can be formed for the parabola.

$$y = k\,|x - a| + b$$

The distance the <u>vertex</u> has moved **vertically** (b)

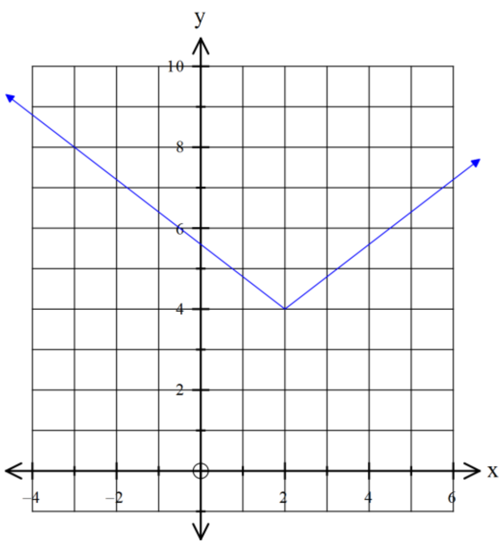
The **scale factor** which makes the absolute value fatter or skinnier (k)

The distance the <u>vertex</u> has moved **horizontally** (a)

If k is negative, then the absolute value will be upside-down and the <u>vertex</u> is the maximum point of the graph.

<u>Example</u>

Write the equation for the following graph.



1. Locate the vertex (2, 4)
2. Substitute the vertex into the general equation:

$$y = k\,|x - 2| + 4$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

Selected point is $(-3, 8)$

$$\begin{aligned} y &= k\,|x - 2| + 4 && \text{Substitute in } (x, y) \\ 8 &= k\,|-3 - 2| + 4 && \text{Subtract 4 from both sides and simplify the brackets} \\ 4 &= k\,|-5| && \text{Take the absolute value of } -5 \\ 4 &= k\,(5) && \text{Divide both sides by 5} \\ \frac{4}{5} &= k \end{aligned}$$

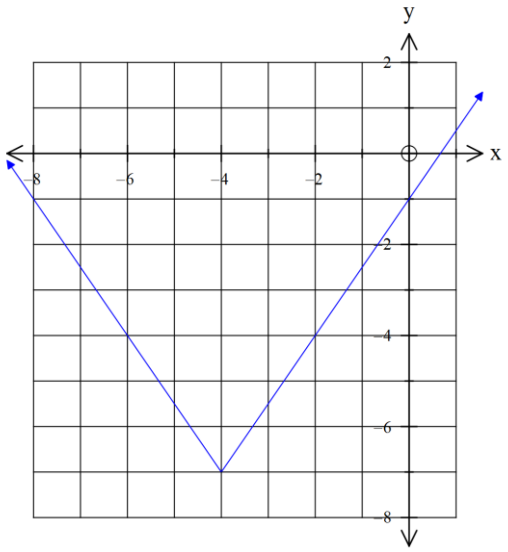
The equation of the graph is $y = \frac{4}{5}|x - 2| + 4$

When selecting the (x, y) co-ordinate to substitute into the equation, you cannot select a point you have already used to form the equation.

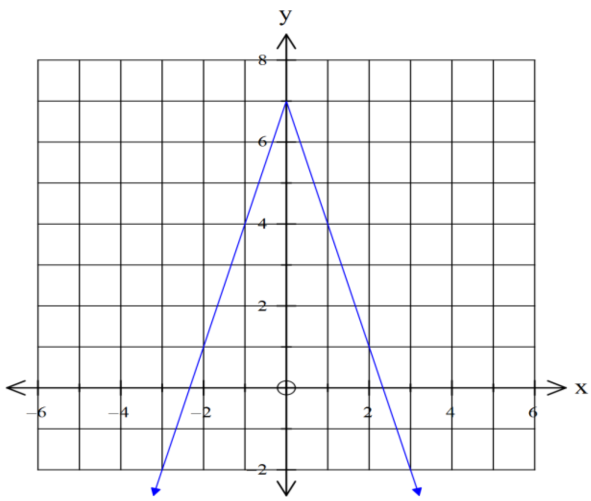
Equations of Absolute Value Graphs

Write the equations of the following absolute value graphs.

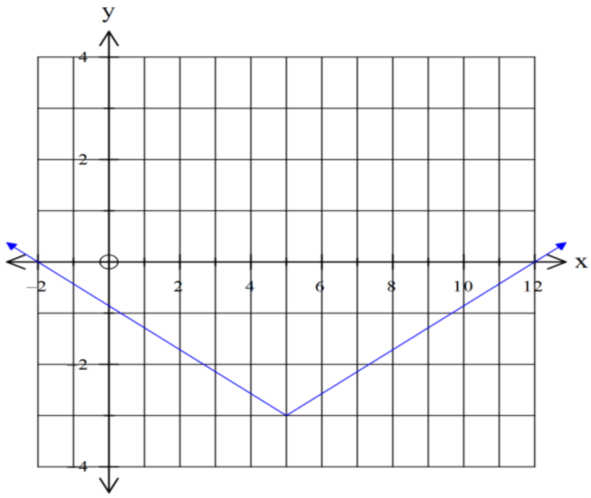
QUESTION ONE



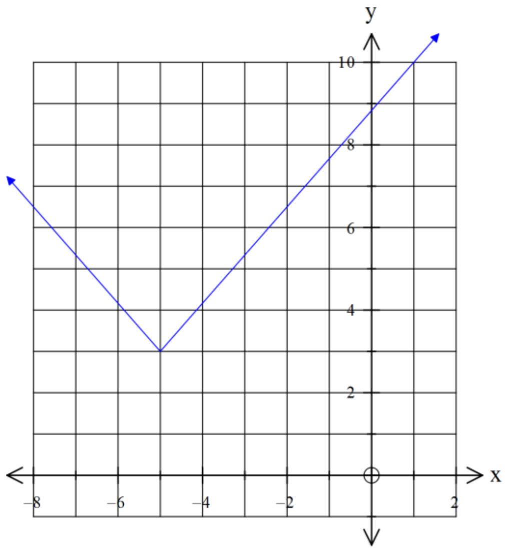
QUESTION TWO



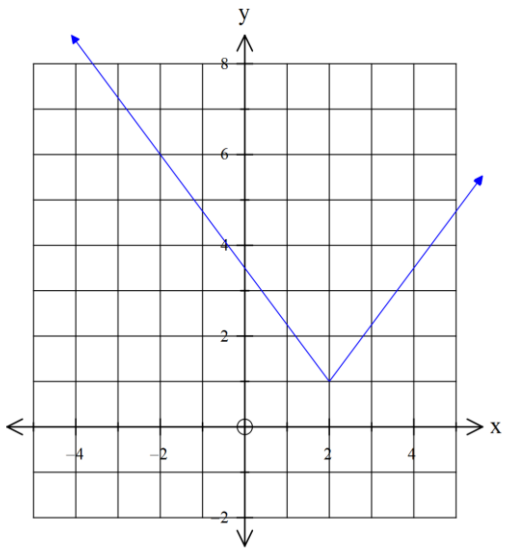
QUESTION THREE



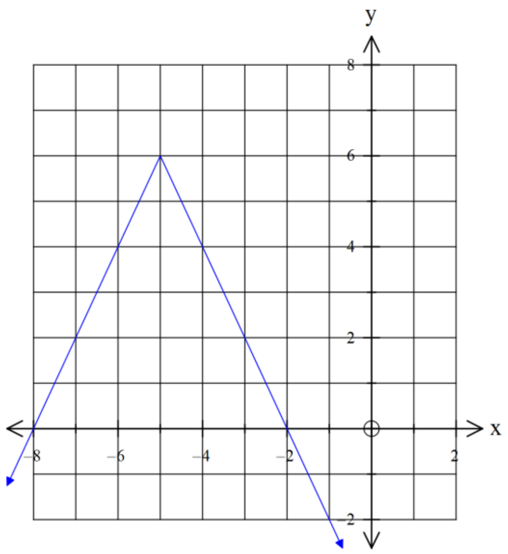
QUESTION FOUR



QUESTION FIVE



QUESTION SIX

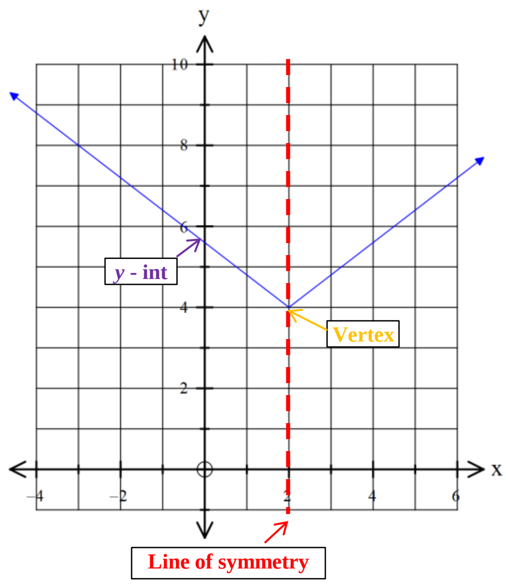


Key Features of Absolute Value Graphs

Example

State the key features for the absolute value graph $y = \frac{4}{5}|x - 2| + 4$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features



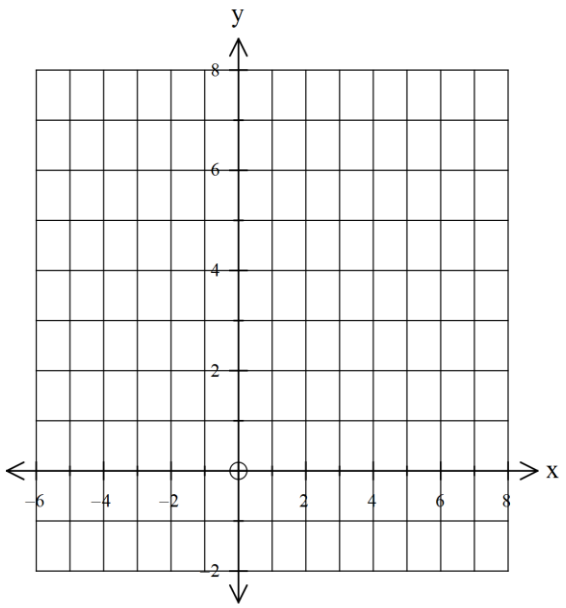
domain	no domain
x – intercept	none
y – intercept	(0, 5.6)
vertex	(2, 4)
line of symmetry	$x = 2$

In this case there is no x-intercept, but there are some absolute value graphs where the x-intercept does exist.

Sketching Absolute Value Graphs

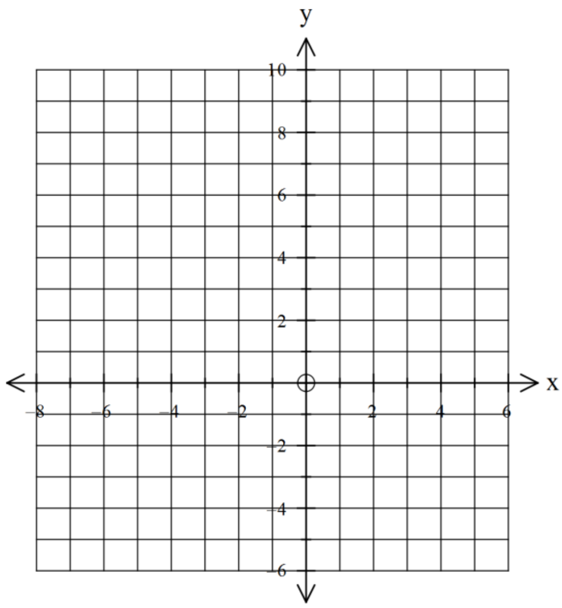
Graph each absolute value graph by finding the key features using your graphics calculators on the grids below.

QUESTION ONE



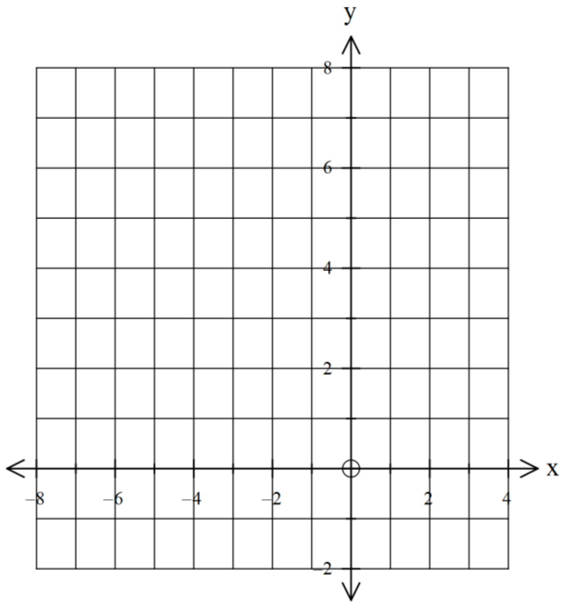
$y = \frac{2}{3}\lvert x - 3\rvert$	
domain	$-3 \le x \le 6$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION TWO



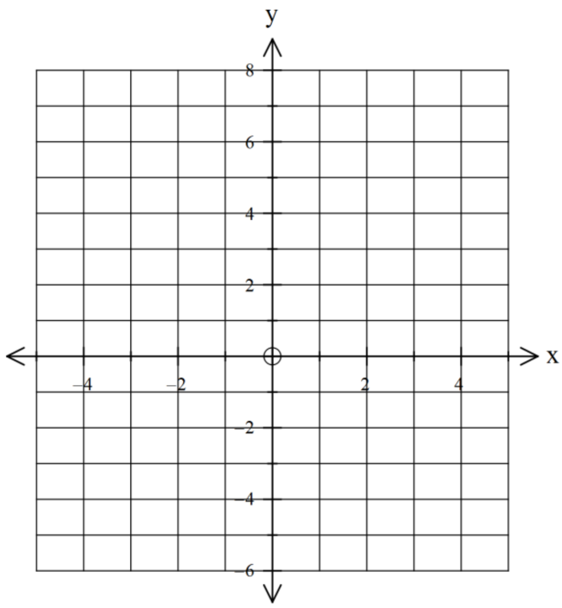
$y = 2\|x + 1\| - 4$	
domain	$-6 \le x \le 2$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION THREE



$y = -\dfrac{3}{2}\|x + 4\| + 6$	
domain	no domain
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION FOUR



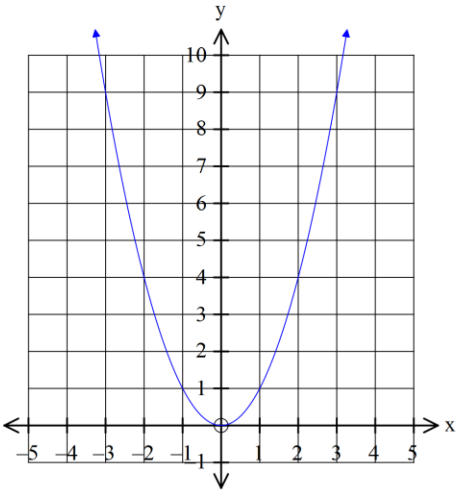
$y = 3\|x\| - 5$	
domain	$x \ge -2$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

Parabolas

The standard parabola is $y = x^2$. It has a vertex (or minimum point) at (0, 0)

$y = x^2$ can be transformed in the following ways:

- $y = x^2 + 2$ shifts $y = x^2$ two units up
- $y = x^2 - 2$ shifts $y = x^2$ two units down
- $y = (x + 2)^2$ shifts $y = x^2$ two units left
- $y = (x - 2)^2$ shifts $y = x^2$ two units right
- $y = 2x^2$ makes $y = x^2$ skinnier
- $y = \frac{1}{2}x^2$ makes $y = x^2$ fatter



If we combine all these transformations, a **general equation** can be formed for the parabola.

$$y = k(x - a)^2 + b$$

The distance the <u>vertex</u> has moved **vertically**

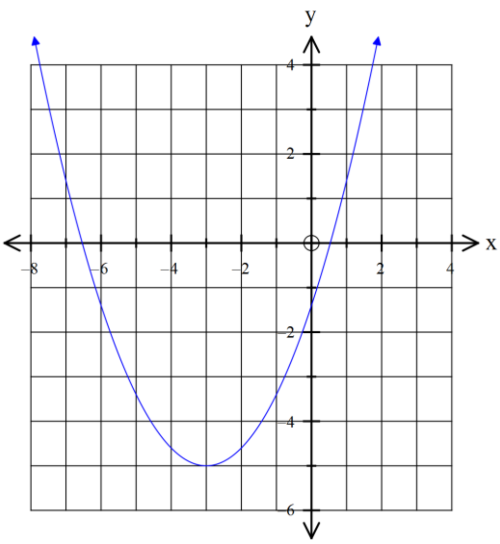
The **scale factor** which makes the parabola fatter or skinnier

The distance the <u>vertex</u> has moved **horizontally**

If k is negative, then the parabola will be upside-down and the <u>vertex</u> is the maximum point of the graph.

<u>Example</u>

Write the equation for the following graph.



1. Locate the vertex $(-3, -5)$

2. Substitute the vertex into the general equation:

$$y = k(x + 3)^2 - 5$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is (2, 5)

$$\begin{aligned} y &= k(x + 3)^2 - 5 && \text{Substitute in } (x, y) \\ 5 &= k(2 + 3)^2 - 5 && \text{Subtract 5 from both sides and simplify the brackets} \\ 10 &= k(5)^2 && \text{Simplify } 5^2 \\ 10 &= k(25) && \text{Divide both sides by 25 and simplify the fraction} \\ \frac{10}{25} &= k = \frac{2}{5} \end{aligned}$$

The equation of the graph is $y = \frac{2}{5}(x + 3)^2 - 5$

The **general equation** for a parabola can also be expressed as

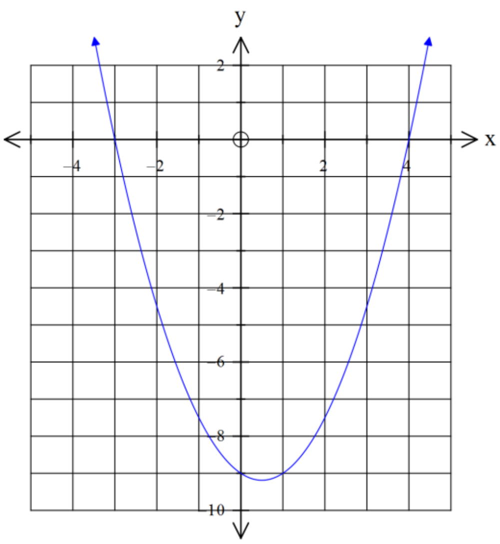
$$y = k(x - c)(x - d)$$

The **scale factor** which makes the parabola fatter or skinnier

The location of the *x*-intercepts

Example

Write the equation for the following graph.



1. Locate the x –intercepts (−3, 0) and (4, 0)

2. Substitute the x –intercepts into the general equation:

$$y = k(x + 3)(x - 4)$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is (0, −9)

$$\begin{aligned} y &= k(x + 3)(x - 4) && \text{Substitute in } (x, y) \\ -9 &= k(0 + 3)(0 - 4) && \text{Simplify brackets} \\ -9 &= k(3)(-4) && \text{Calculate } 3 \times -4 \\ -9 &= k(-12) && \text{Divide both sides by } -12 \text{ and simplify} \\ \frac{9}{12} &= k = \frac{3}{4} \end{aligned}$$

The equation of the graph is $y = \frac{3}{4}(x + 3)(x - 4)$

Parabolic Summary

When the vertex is known, use

$$y = k(x - a)^2 + b$$

The distance the vertex has moved **vertically**

The **scale factor** which makes the parabola fatter or skinnier

The distance the vertex has moved **horizontally**

When the *x*-intercepts is known, use

$$y = k(x - c)(x - d)$$

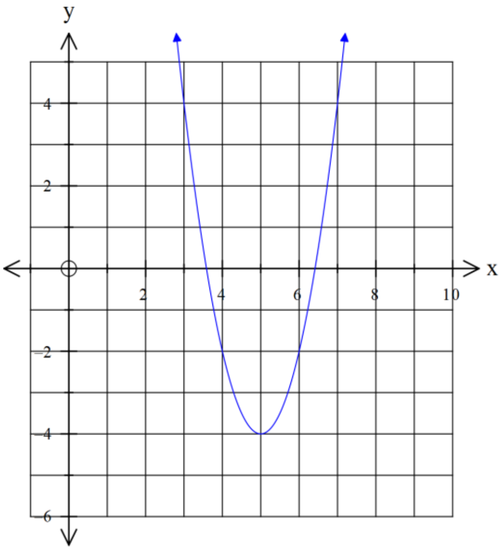
The **scale factor** which makes the parabola fatter or skinnier

The location of the *x*-intercepts

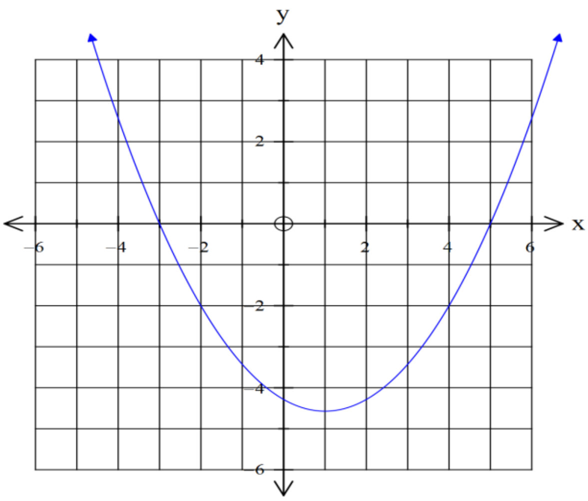
Equations of Parabolas

Write the equations of the following parabolas.

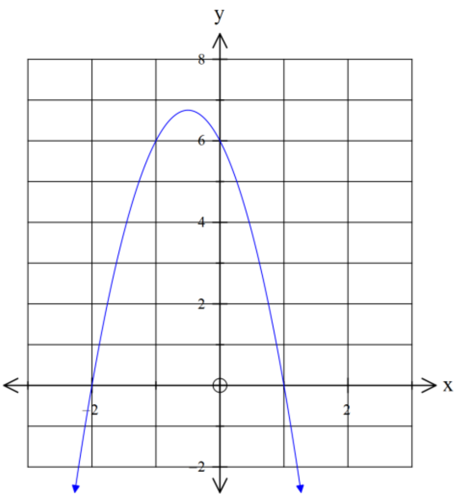
QUESTION ONE



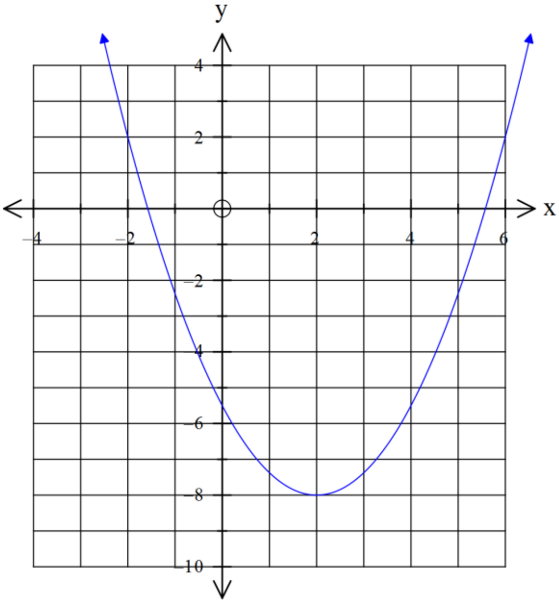
QUESTION TWO



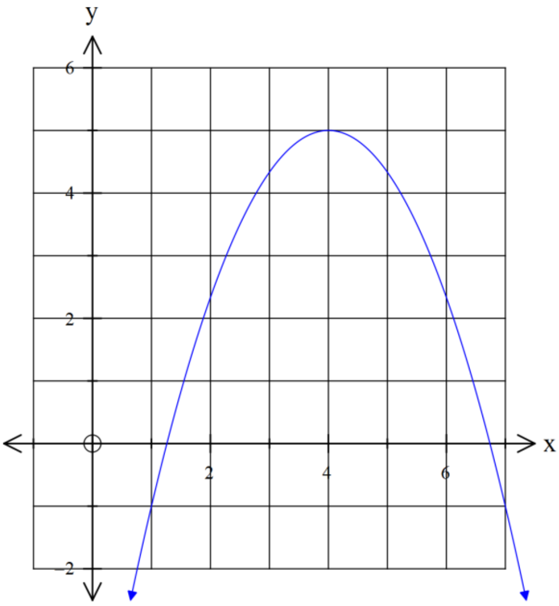
QUESTION THREE



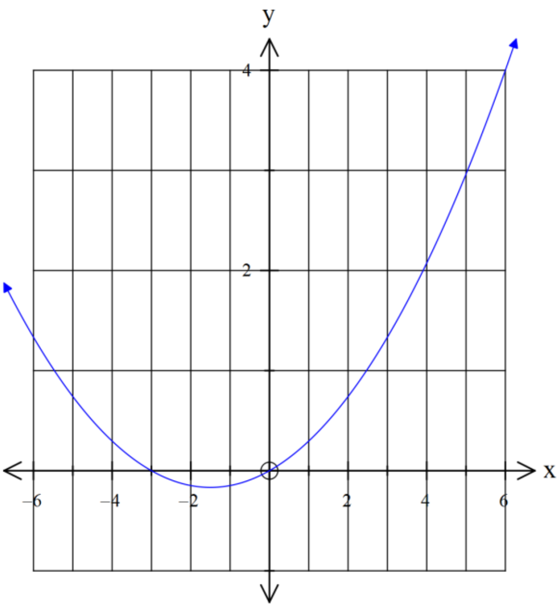
QUESTION FOUR



QUESTION FIVE



QUESTION SIX



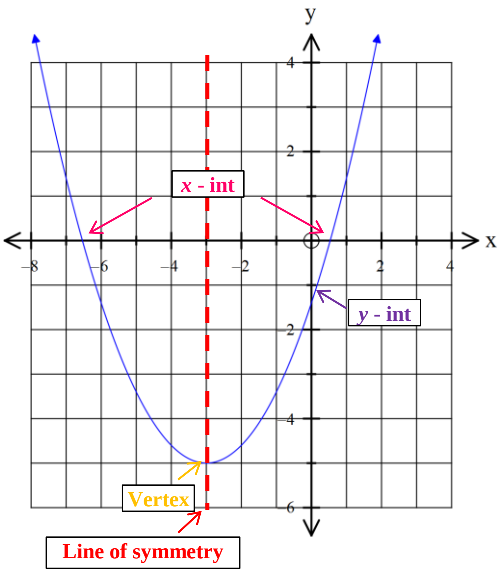
Key Features of Parabolas

Example

State the key features for the parabola $y = \frac{2}{5}(x+3)^2 - 5$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features

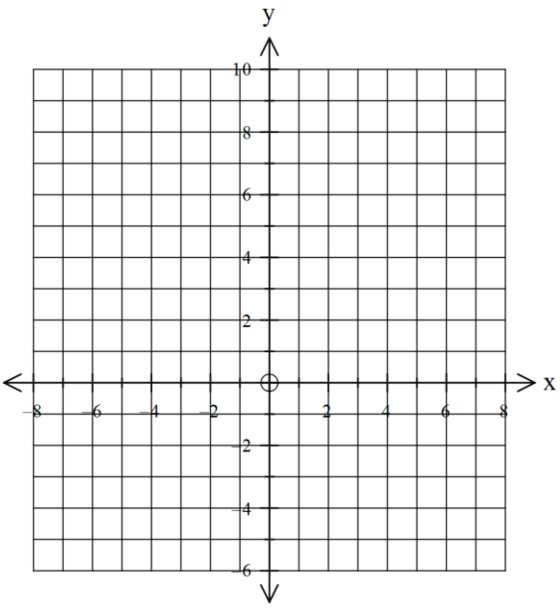
domain	no domain
x – intercept	$(-6.54, 0)$ and $(0.534, 0)$
y – intercept	$(0, 1.4)$
vertex	$(-3, -5)$
line of symmetry	$x = -3$



Sketching Parabolas

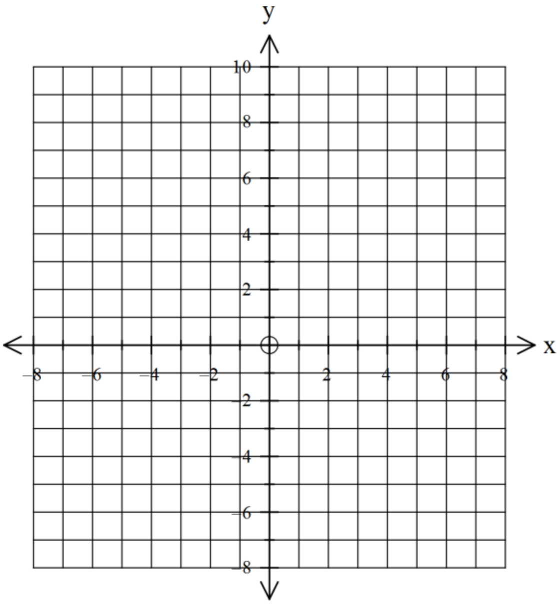
Graph each parabola by finding the key features using your graphics calculators on the grids below.

QUESTION ONE



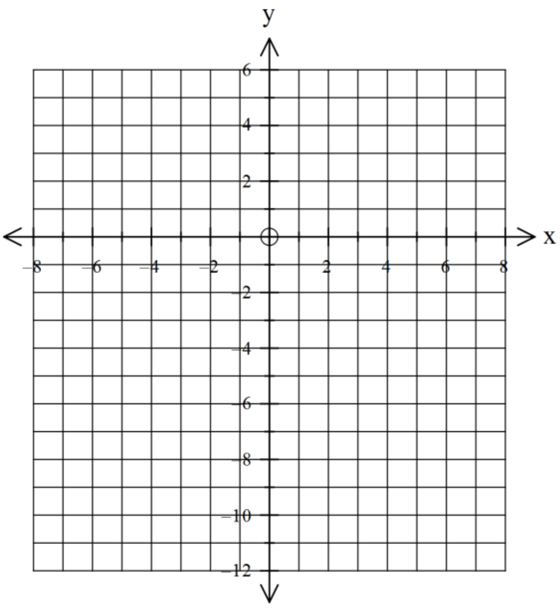
$y = 3(x-3)^2$	
domain	$-4 \le x \le 6$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION TWO



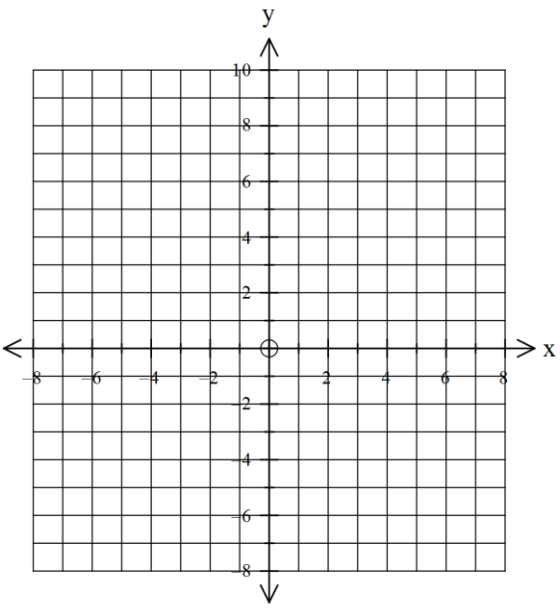
$y = -2\,(x-3)\,(x+1)$	
domain	$-1 \le x \le 2$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION THREE



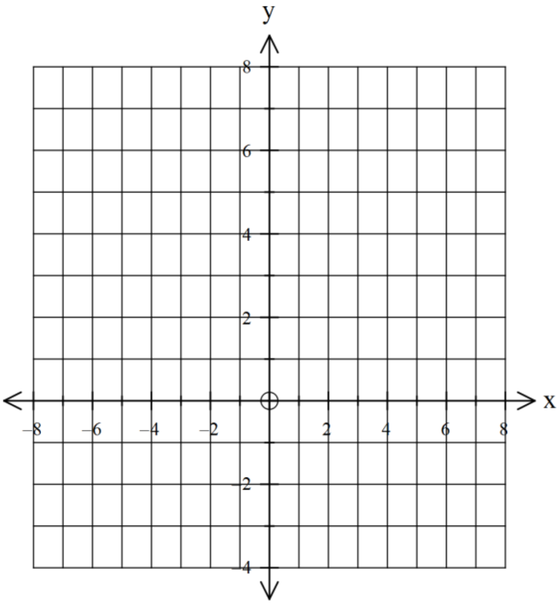
$y = \frac{2}{7}(x+5)(x-7)$	
domain	no domain
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION FOUR



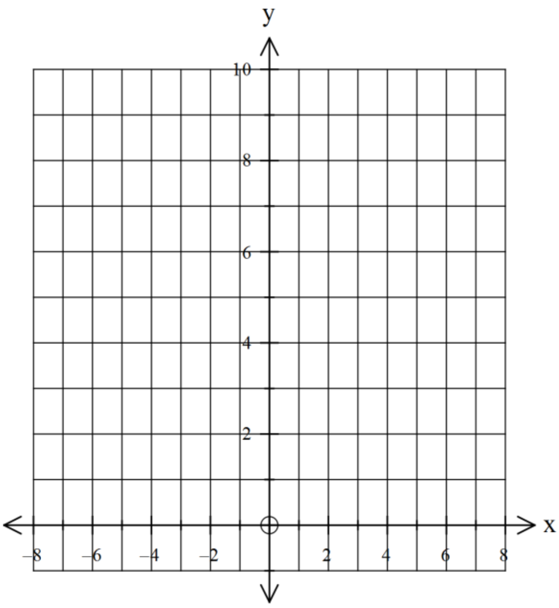
$y = -\frac{1}{4}(x-2)^2 + 8$	
domain	$x \ge -4$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION FIVE



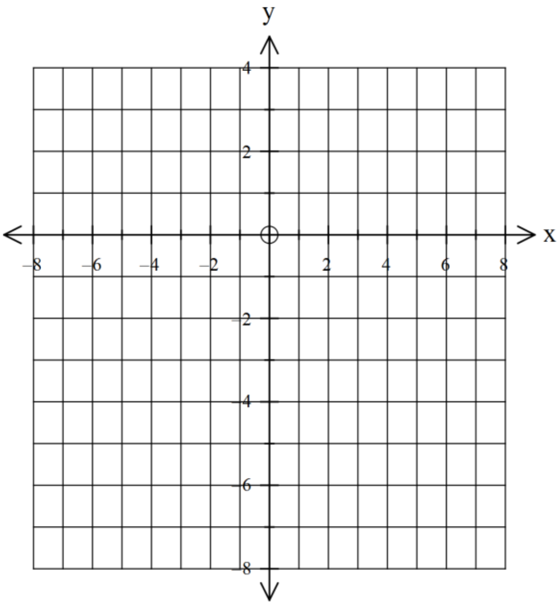
$y = -\frac{2}{5}(x-1)(x+5)$	
domain	$-4 \le x \le 1$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

QUESTION SIX



$y = 2(x+5)^2 + 2$	
domain	no domain
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

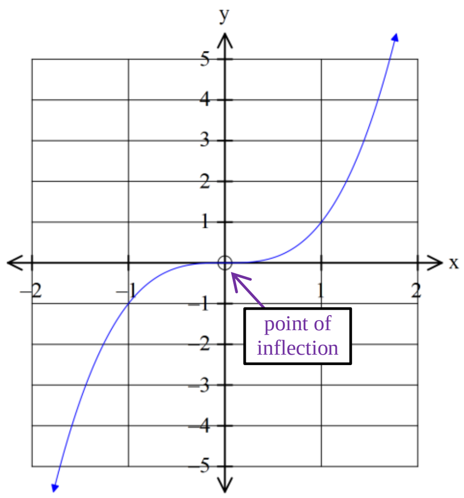
QUESTION SEVEN



$y = 3(x+1)\,(x-2)$	
domain	$x \le 2$
x – intercept(s)	
y – intercept	
vertex	
line of symmetry	

Cubics

The standard cubic is $y = x^3$. It has a point of inflection at (0, 0)



A cubic can also be described as a polynomial of degree 3.

It is from the same family as a parabola, which is a polynomial of degree 2.

The **general equation** where the standard cubic has been transformed is given as

$$y = k\,(x - a)^3 + b$$

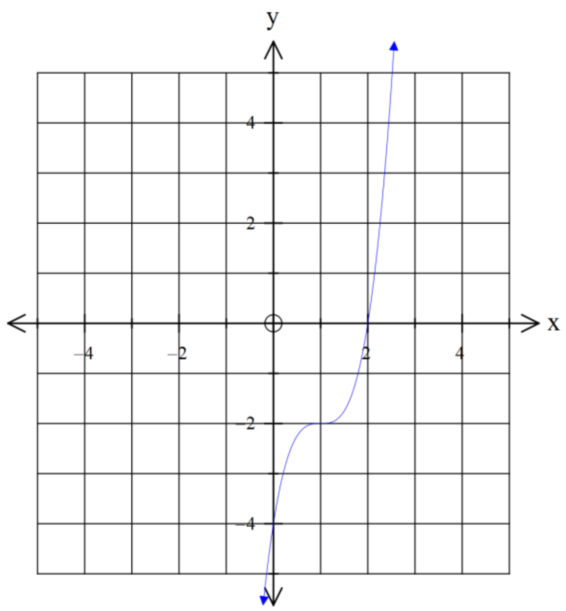
The distance the point of inflection has moved **vertically**

The **scale factor** which makes the cubic fatter or skinnier

The distance the point of inflection has moved

Example

Write the equation for the following graph.



1. Locate the point of inflection (1, −2)

2. Substitute the point of inflection into the general equation:

$$y = k\,(x - 1)^3 - 2$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is (0, −4)

$$\begin{aligned} y &= k\,(x - 1)^3 - 2 && \text{Substitute in } (x, y) \\ -4 &= k\,(0 - 1)^3 - 2 && \text{Add 2 to both sides and simplify the brackets} \\ -2 &= k\,(-1)^3 && \text{Simplify } (-1)^3 \\ -2 &= k\,(-1) && \text{Divide both sides by } -1 \\ 2 &= k \end{aligned}$$

The equation of the graph is $y = 2(x - 1)^3 - 2$

The **general equation** for a cubic can also be expressed as

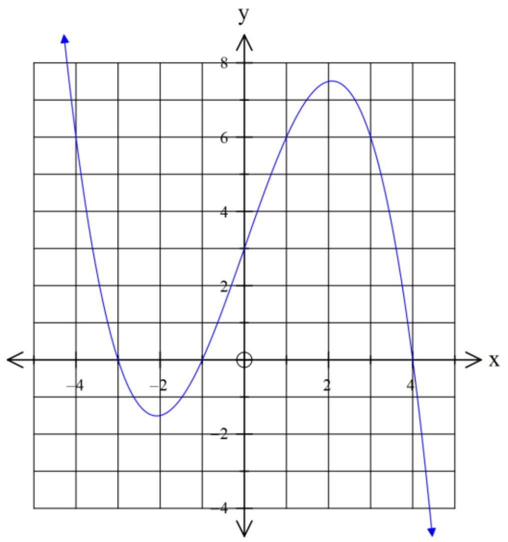
$$y = k(x - c)(x - d)(x - e)$$

The **scale factor** which makes the cubic fatter or skinnier

The location of the *x*-intercepts

Example

Write the equation for the following graph.



1. Locate the x –intercepts $(-3, 0)$, $(-1, 0)$ and $(4, 0)$

2. Substitute the x –intercepts into the general equation:

$$y = k(x + 3)(x + 1)(x - 4)$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is $(0, 3)$

$$y = k(x + 3)(x + 1)(x - 4) \quad \text{Substitute in } (x, y)$$
$$3 = k(0 + 3)(0 + 1)(0 - 4) \quad \text{Simplify brackets}$$
$$3 = k(3)(1)(-4) \quad \text{Calculate } 3 \times 1 \times -4$$
$$3 = k(-12) \quad \text{Divide both sides by } -12 \text{ and simplify}$$
$$-\frac{3}{12} = k = -\frac{1}{4}$$

The equation of the graph is $y = -\frac{1}{4}(x + 3)(x + 1)(x - 4)$

Cubic Summary

When the point of inflection is known, use

$$y = k(x - a)^3 + b$$

The **scale factor** which makes the cubic fatter or skinnier

The distance the point of inflection has moved **horizontally**

The distance the point of inflection has moved **vertically**

When the *x-intercepts* is known, use

$$y = k(x - c)(x - d)(x - e)$$

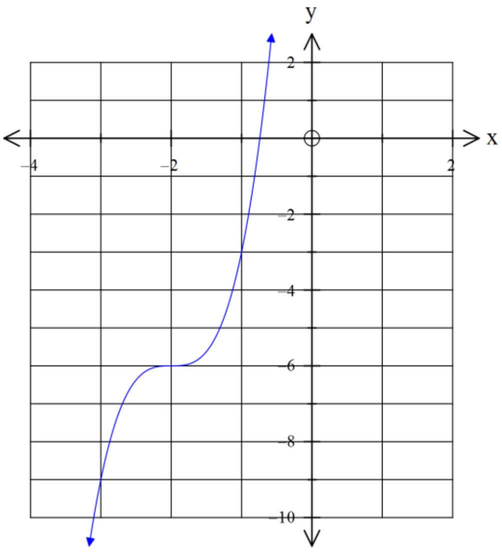
The **scale factor** which makes the cubic fatter or skinnier

The location of the *x*-intercepts

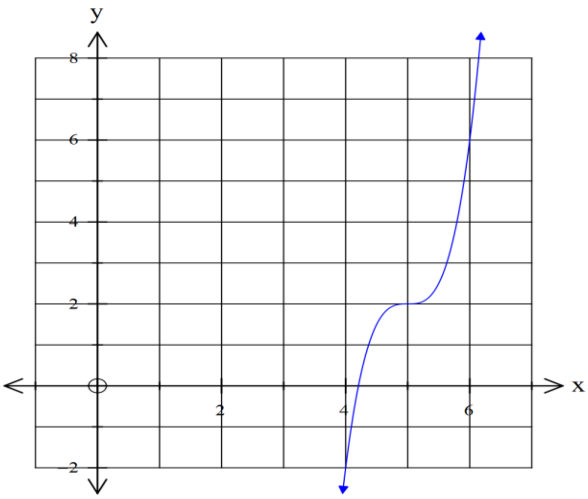
Equations of Cubics

Write the equations of the following cubics.

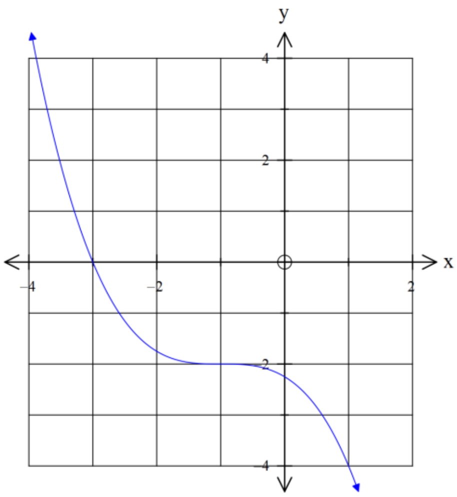
QUESTION ONE



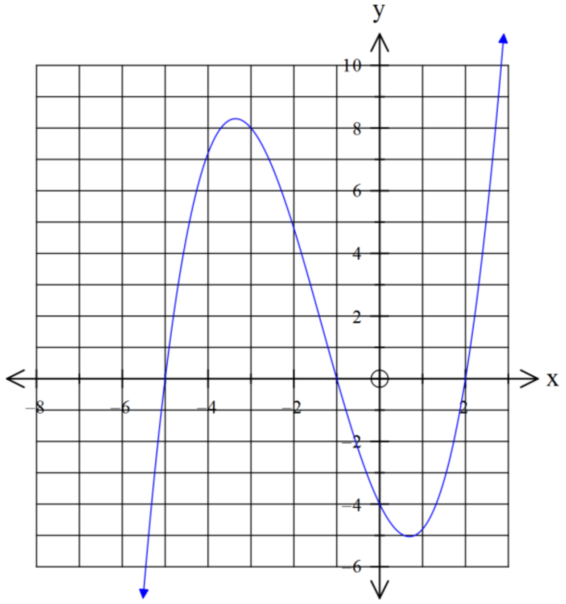
QUESTION TWO



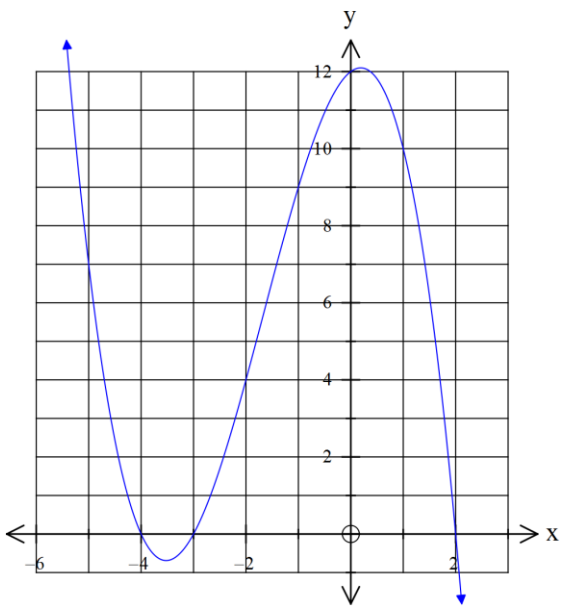
QUESTION THREE



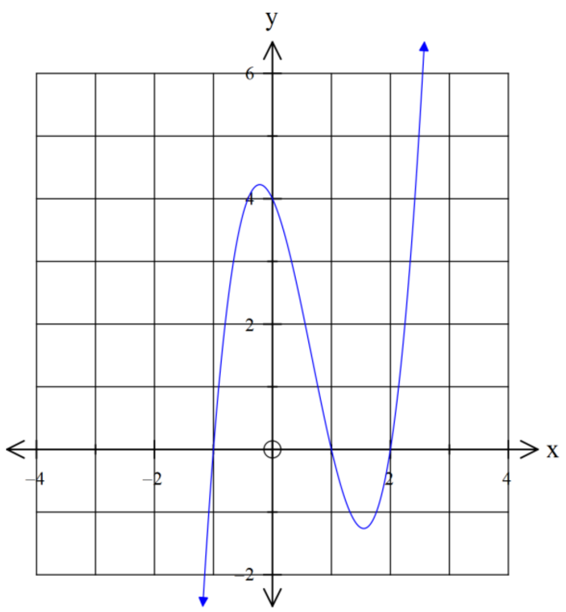
QUESTION FOUR



QUESTION FIVE



QUESTION SIX

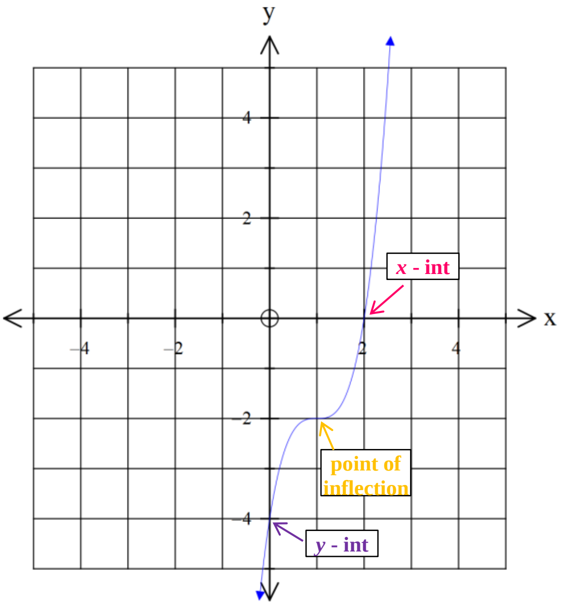


Key Features of Cubics

Example

State the key features for the cubic $y = 2(x - 1)^3 - 2$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features



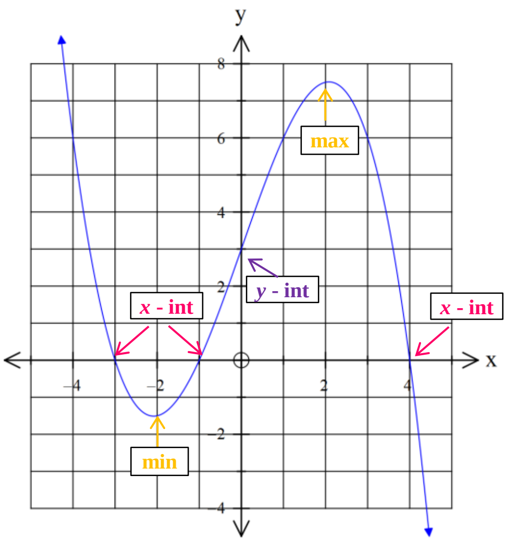
domain	no domain
x – intercept	(2, 0)
y – intercept	(0, −4)
point of inflection	(1, −2)

Note that the calculator cannot identify the point of inflection, you need to be able to do this yourself.

Example

State the key features for the cubic $y = -\frac{1}{4}(x + 3)(x + 1)(x - 4)$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features

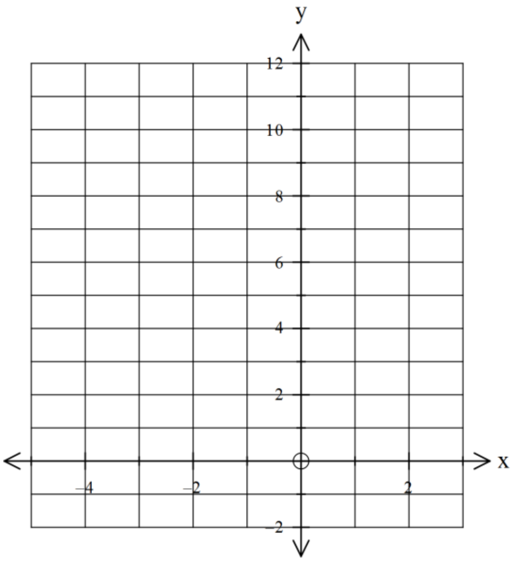


domain	no domain
x – intercept	(−3, 0), (−1, 0) and (4, 0)
y – intercept	(0, 3)
maximum	(2.08, 7.51)
minimum	(−2.08, −1.51)

Sketching Cubics

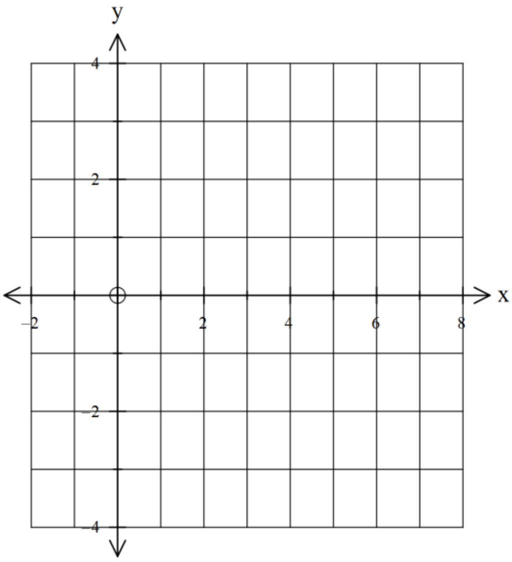
Graph each cubic by finding the key features using your graphics calculators on the grids below.

QUESTION ONE



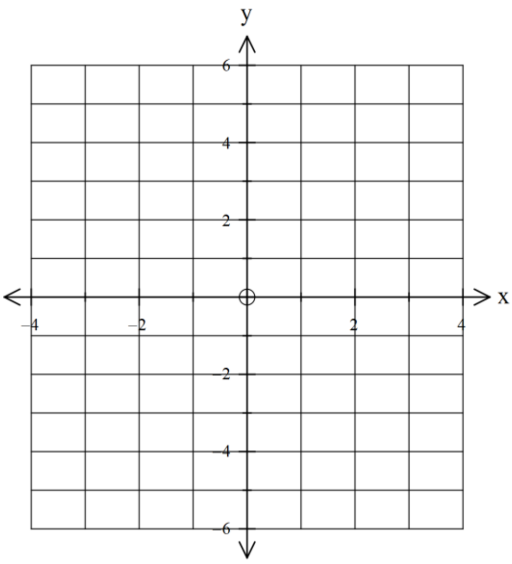
$y = \frac{3}{4}(x+2)^3 + 5$	
domain	$x \leq 0$
x – intercept(s)	
y – intercept	
point of inflection	

QUESTION TWO



$y = -2(x-4)^3 + 1$	
domain	no domain
x – intercept(s)	
y – intercept	
point of inflection	

QUESTION THREE



$y = \frac{6}{5}(x+1)(x-1)(x-2)$	
domain	$-1 \leq x \leq 2$
x – intercept(s)	
y – intercept	
maximum	
minimum	

Higher Powered Polynomials

The parabola and cubic belong to the polynomial family.

Higher powered polynomials can have powers of degree 3 or higher on the unknown x.

The **general equation** for a higher powered polynomial can also be expressed as

$$y = k\,(x - c)(x - d)(x - e)(x - f)\ldots$$

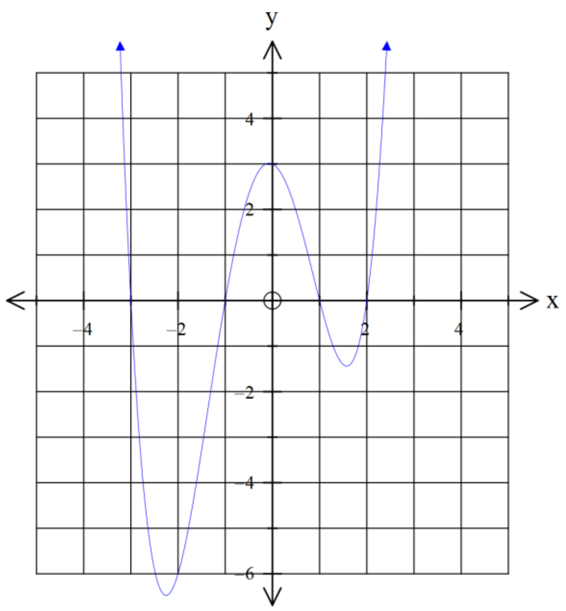
The **scale factor** which makes the polynomial fatter or skinnier

The location of the x-intercepts

The number of brackets we use depends on the degree of the polynomial (power 4 needs 4 brackets etc.)

Example

Write the equation for the following graph.



1. Locate the x-intercepts $(-3, 0)$, $(-1, 0)$, $(1, 0)$ and $(2, 0)$

2. Substitute the x – intercepts into the general equation:

$$y = k\,(x + 3)(x + 1)(x - 1)(x - 2)$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

Selected point is $(0, 3)$

$y = k\,(x + 3)(x + 1)(x - 1)(x - 2)$ Substitute in (x, y)
$3 = k\,(0 + 3)(0 + 1)(0 - 1)(0 - 2)$ Simplify brackets
$3 = k\,(3)(1)(-1)(-2)$ Simplify $3 \times 1 \times -1 \times -2$
$3 = k\,(6)$ Divide both sides by 6
$0.5 = k$

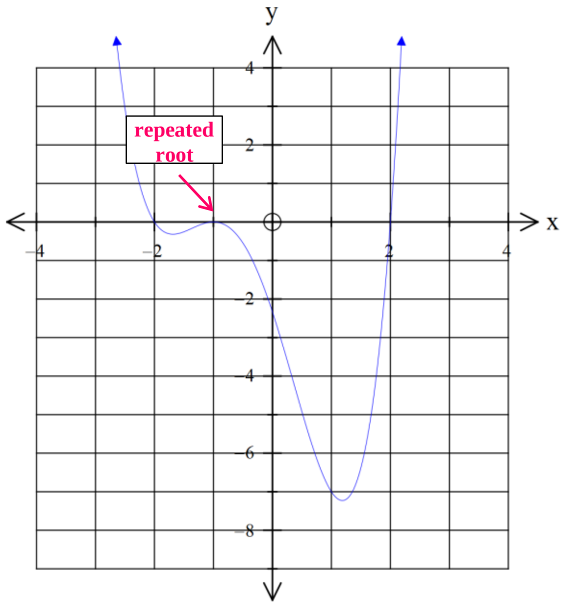
The equation of the graph is $y = \frac{1}{2}(x + 3)(x + 1)(x - 1)(x - 2)$

Higher Powered Polynomials: The Repeated Root

If the graph bounces off the x – axis, then there is a repeated root.

Example

Write the equation for the following graph.



1. Locate the x-intercepts $(-2, 0)$, $(-1, 0)$, $(-1, 0)$ and $(2, 0)$
2. Substitute the x – intercepts into the general equation:

$$y = k\,(x + 2)\,(x + 1)^2\,(x - 2)$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is $(1, -7)$

$$\begin{aligned} y &= k\,(x + 2)\,(x + 1)^2\,(x - 2) \\ -7 &= k\,(1 + 2)\,(1 + 1)^2\,(1 - 2) \\ -7 &= k\,(3)(2)^2\,(-1) \\ -7 &= k\,(-12) \\ \frac{7}{12} &= k \end{aligned}$$

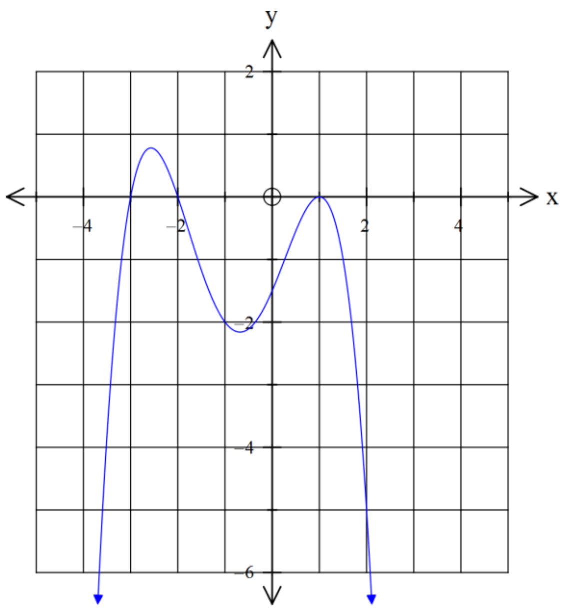
Substitute in (x, y)
Simplify brackets
Multiply $3 \times 4 \times -1$

The equation of the graph is $y = \frac{7}{12}\,(x + 2)(x + 1)^2(x - 2)$

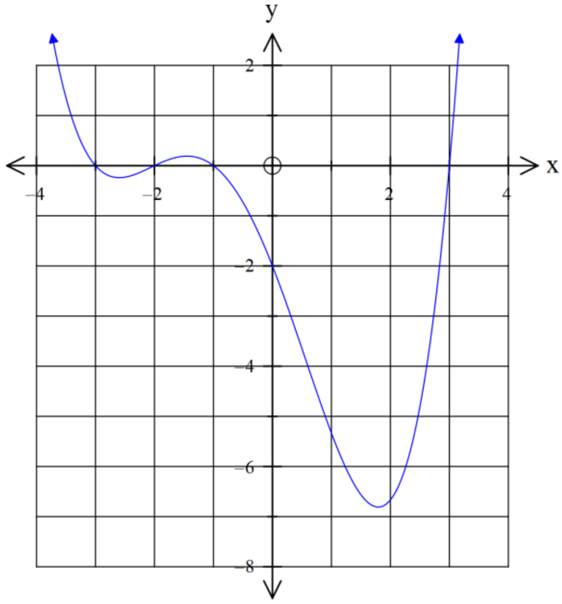
Equations of Higher Powered Polynomials

Write the equations of the following higher powered polynomials

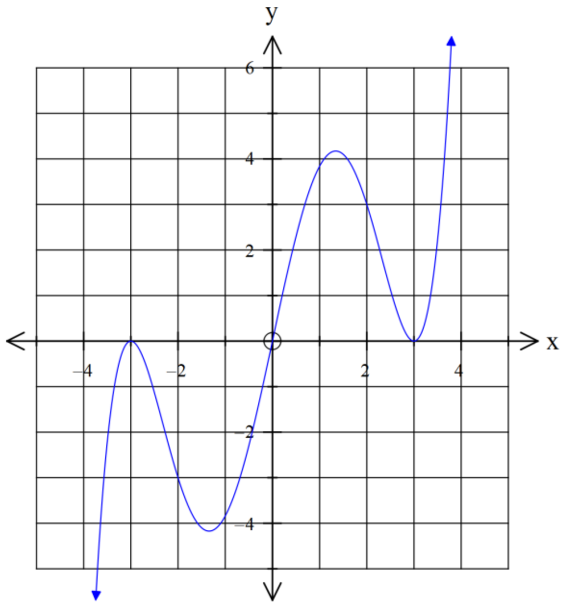
QUESTION ONE



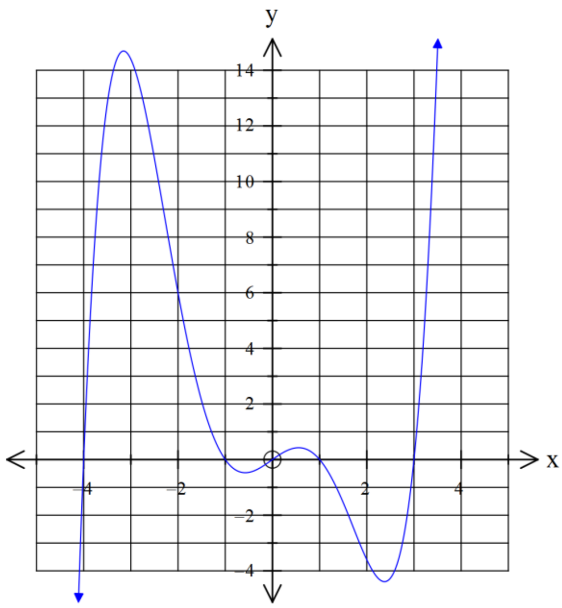
QUESTION TWO



QUESTION THREE



QUESTION FOUR

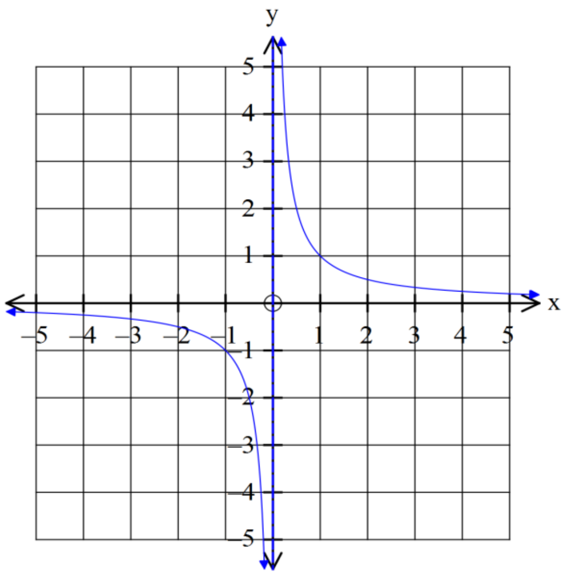


Rectangular Hyperbolae

The standard hyperbola is $y = \frac{1}{x}$. It has asymptotes at $x = 0$ and $y = 0$

The asymptote is a point or line that the graph approaches but will never touch. The asymptotes are usually not drawn on the graph.

An asymptote exists for the hyperbola as dividing by zero is a mathematical impossibility.



x	$y = \frac{1}{x}$
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$

The **general equation** where the standard hyperola has been transformed is given as

The **scale factor** which makes the hyperbola fatter or skinnier

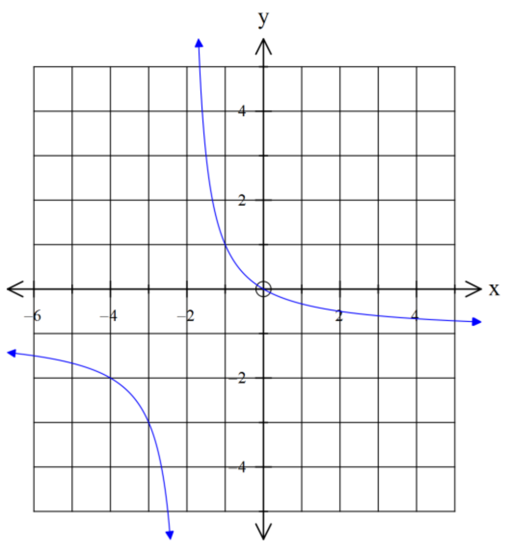
$$y = \frac{k}{x-a} + b$$

The distance the horizontal asymptote has moved **vertically**

The distance the vertical asymptote has moved **horizontally**

Example

Write the equation for the following graph.



1. Locate the asymptotes
   Horizontal asymptote: $y = -1$
   Vertical asymptote: $x = -2$

2. Substitute the asymptotes into the general equation:

$$y = \frac{k}{x+2} - 1$$

3. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is $(-1, 1)$

$$y = \frac{k}{x+2} - 1$$

Substitute in (x, y)

$$1 = \frac{k}{-1+2} - 1$$

Simplify $-1 + 2$

$$1 = \frac{k}{1} - 1$$

Add 1 to both sides

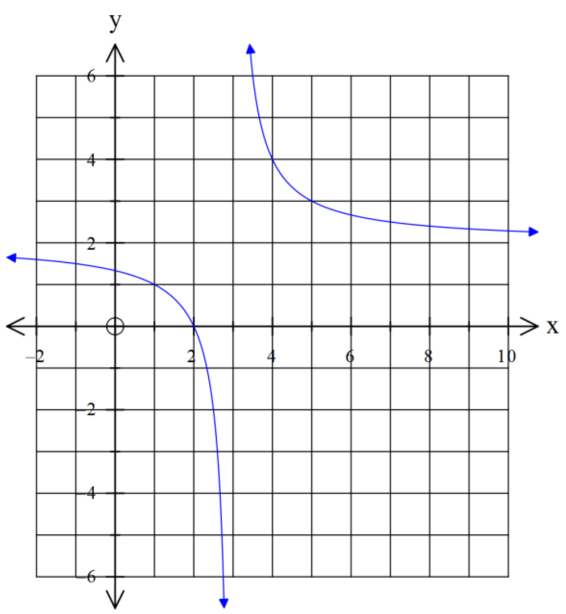
$$2 = k$$

The equation of the graph is $y = \frac{2}{x+2} - 1$

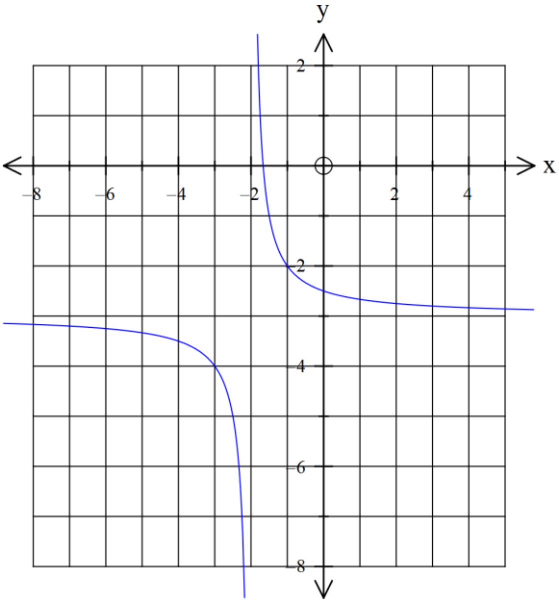
Equations of Rectangular Hyperbolae

Write the equations of the following hyperbolas

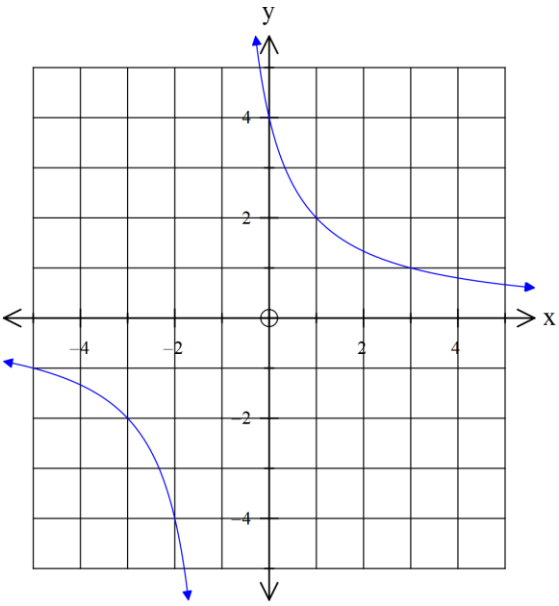
QUESTION ONE



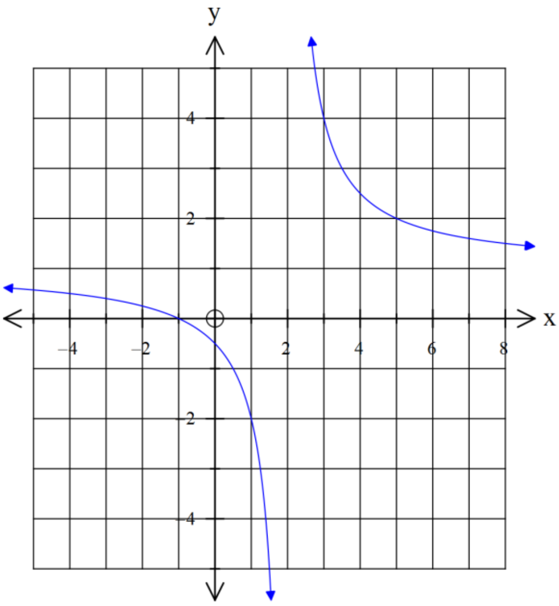
QUESTION TWO



QUESTION THREE



QUESTION FOUR



Key Features of Rectangular Hyperbolae

<u>Example</u>

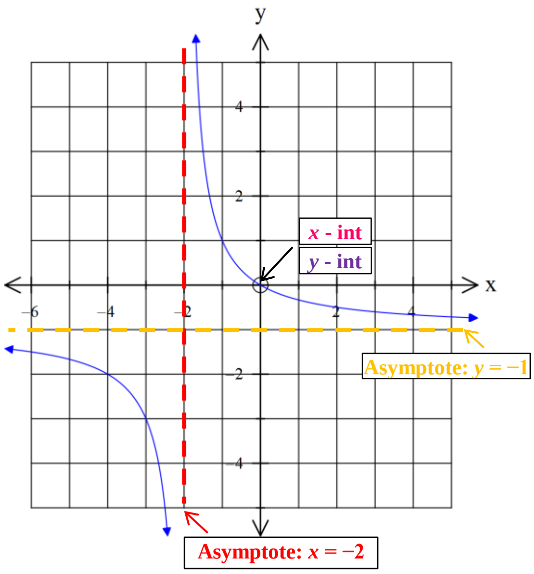
State the key features for the hyperbolas $y \;= \frac{2}{x+2} - 1$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features

domain	no domain
x – intercept	(0, 0)
y – intercept	(0, 0)
vertical asymptote	$x = -2$
horizontal asymptote	$y = -1$

The x – intercept and y – intercept happen to be the same in this example, but this is not always the case.

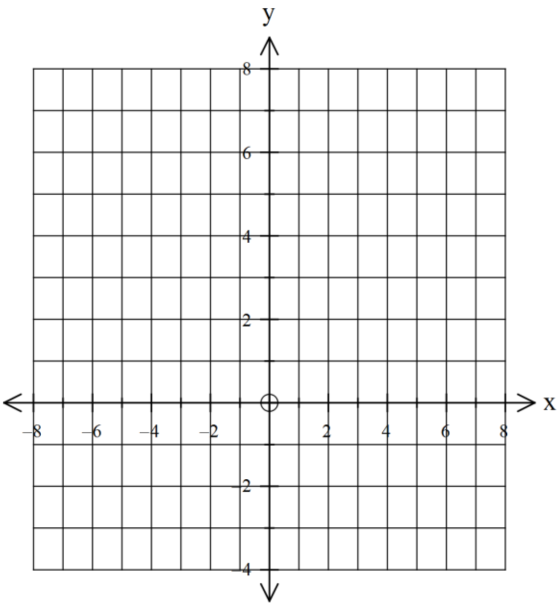
Note that the calculator cannot identify the point of inflection, you need to be able to do this yourself. The asymptotes are also typically not included in the graph.



Sketching Rectangular Hyperbolae

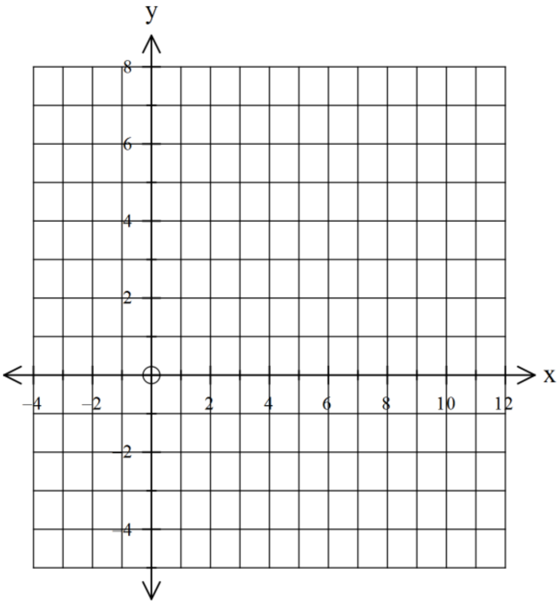
Graph each hyperbola by finding the key features using your graphics calculators on the grids below.

QUESTION ONE



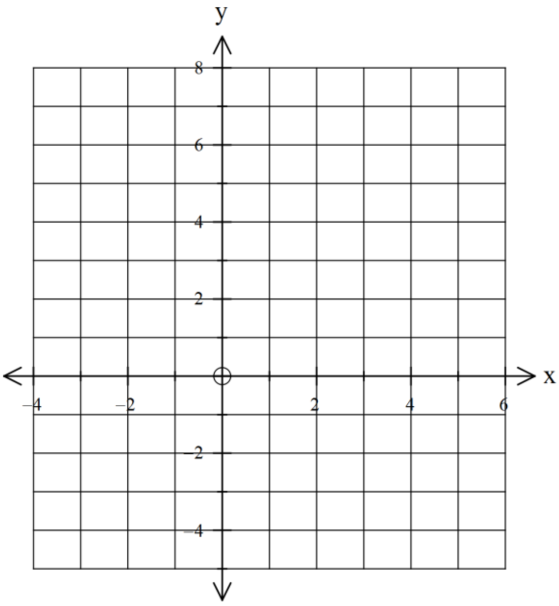
$y = 2 + \dfrac{2}{x-1}$	
domain	$x \geq -1$
x – intercept(s)	
y – intercept	
vertical asymptote	
horizontal asymptote	

QUESTION TWO



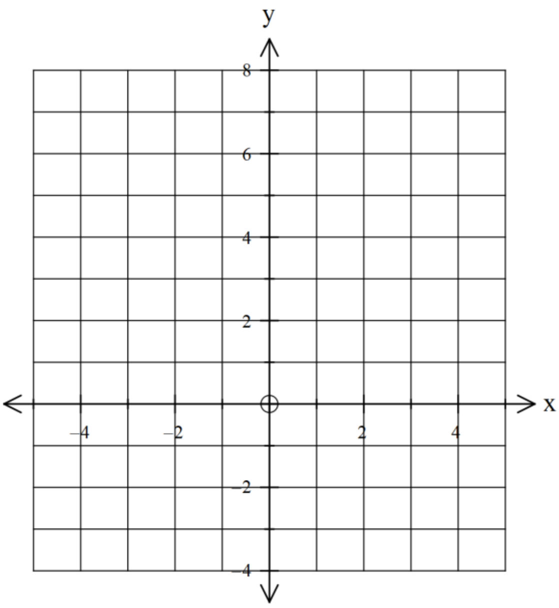
$y = \dfrac{7}{x-4} + 2$	
domain	$-3 \le x \le 6$
x – intercept(s)	
y – intercept	
vertical asymptote	
horizontal asymptote	

QUESTION THREE



$y = -\dfrac{3}{x-1}$	
domain	no domain
x – intercept(s)	
y – intercept	
vertical asymptote	
horizontal asymptote	

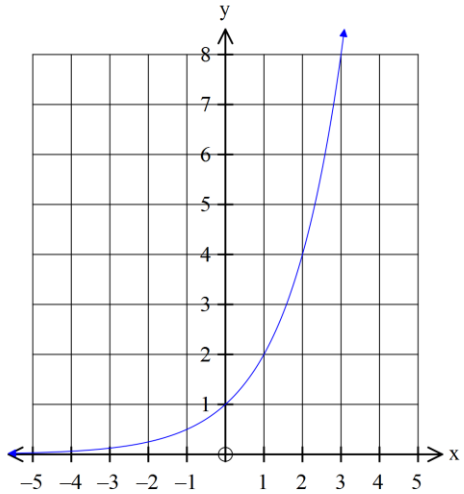
QUESTION FOUR



$y = 3 - \dfrac{1}{x}$	
domain	$x \le 2$
x – intercept(s)	
y – intercept	
vertical asymptote	
horizontal asymptote	

Exponential Function

The standard exponential is $y = p^x$ where p is a constant. It has an <u>asymptote</u> at $y = 0$.



x	$y = 2^x$
-3	$2^{-3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	2
2	4
3	8

Any value to the power of zero equals 1. This occurs as there is a pattern of dividing by 2 in the right hand column of this table

An asymptote exists on the x-axis only, as the more negative the x-values, the smaller its corresponding y-value – but this y-value although very close to 0 will never reach zero.

Since anything to the power of zero equals 1, the standard exponential will always pass through the co-ordinate (0, 1).

The **general equation** for an exponential can be expressed as

$$y = p^{x-a} + b$$

A constant that can make the graph steeper

The distance the <u>asymptote</u> has moved **vertically**

Select the point one unit above the asymptote. The distance this point is from the y-axis is the **horizontal movement**

The flowchart on the right can be used to assist in writing the equation of the exponential graph

Is the asymptote on the x-axis?

Yes

$b = 0$

No

Locate asymptote. This is b.

Find the point 1 unit up from the asymptote. Is this point on the y – axis?

Yes

$c = 0$

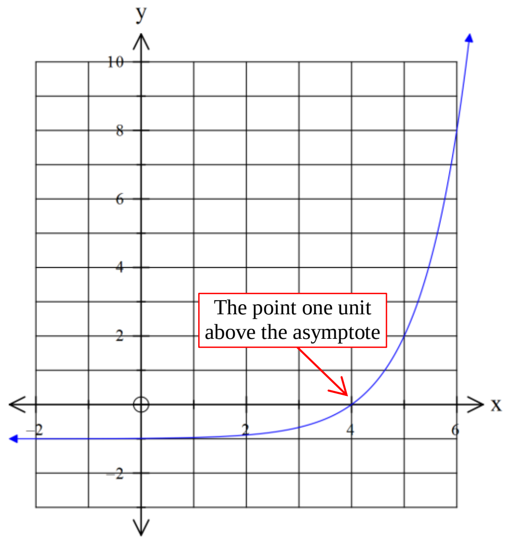
No

Count how far away this point is from the y – axis. This is a.

Sub in known (x, y) to find p

Example

Write the equation for the following graph.



1. Locate the asymptote Horizontal asymptote: $y = -1$

2. Locate the point one unit above the asymptote
The point is $+4$ units away from the y – axis

3. Substitute the values of a and b into the equation

$$y = p^{x-4} - 1$$

4. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

Selected point is $(5, 2)$

$y = p^{x-4} - 1$ Substitute in (x, y)
$2 = p^{5-4} - 1$ Simplify $5 - 4$
$2 = p^1 - 1$ Add 1 to both sides
$3 = p$

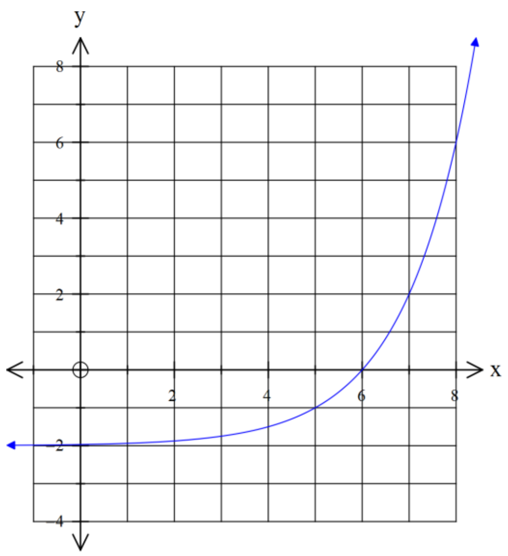
The equation of the graph is $y = 3^{x-4} - 1$

Strategically select your (x, y) co-ordinates so that you have as little rearranging to do as possible. If the power on p is not equal to 1, you will need to take the root of both sides to get p by itself.

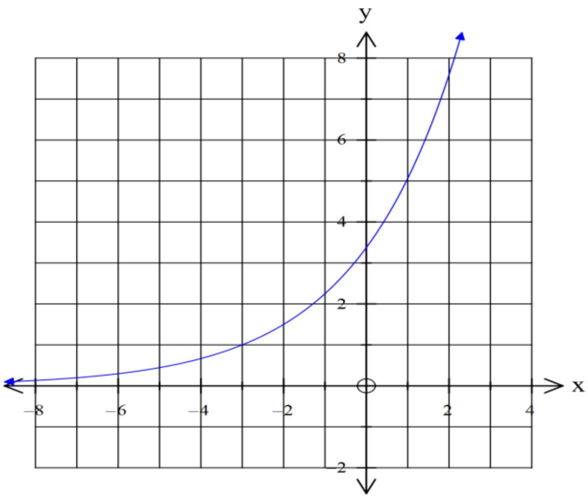
Equations of Exponential Functions

Write the equations of the following exponential functions

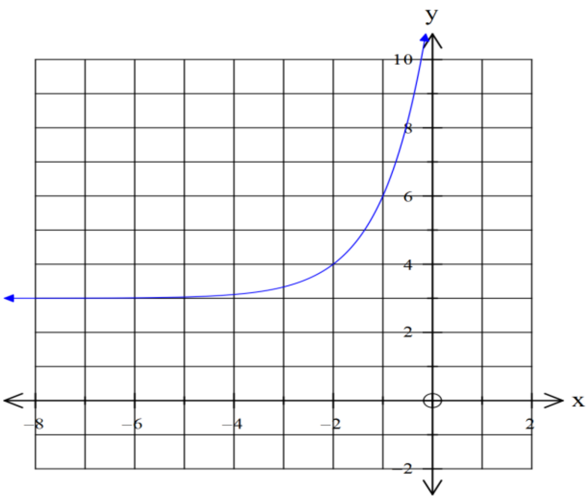
QUESTION ONE



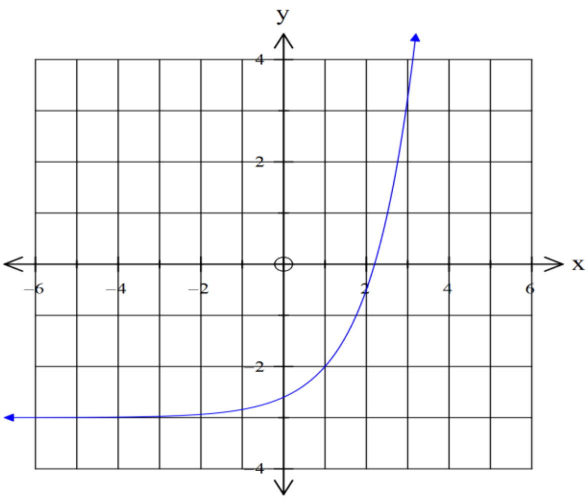
QUESTION TWO



QUESTION THREE



QUESTION FOUR



Exponential Function: Decay and Growth Curves

The **general equation** for an exponential can be expressed as

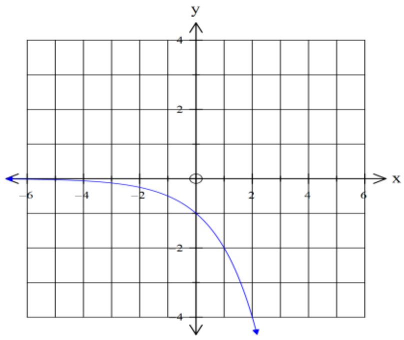
$$y = p^{x-a} + b$$

The distance the <u>asymptote</u> has moved **vertically**

A constant that can make the graph steeper

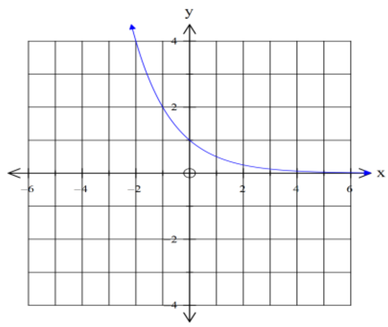
Select the point one unit above the asymptote. The distance this point is from the y-axis is the **horizontal movement**

For different values of p, the exponential graph can look quite different:



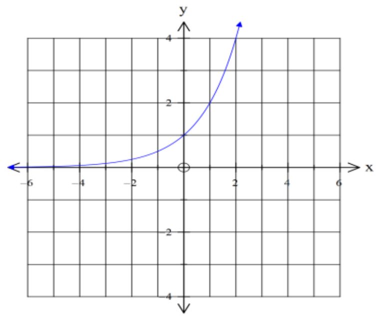
$y = -2^x$

If p is less than -1, then the graph slopes downwards



$y = \frac{1}{2}^x$

If p is between 0 and 1, then the graph is a decay curve



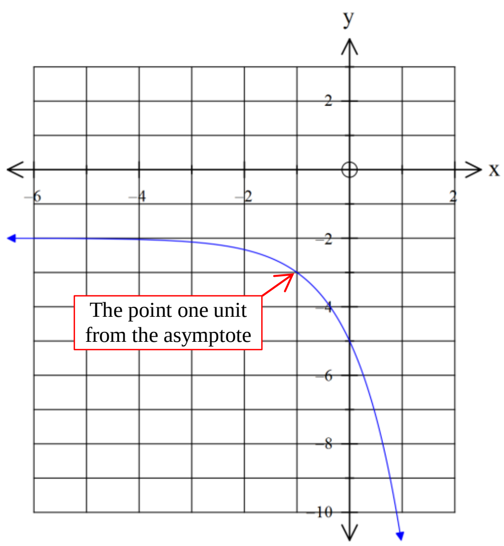
$y = 2^x$

If p is greater than 1, then the graph is a growth curve

The exponential will never have $p = 1$ or $p = 0$, as 1 to the power of any value is 1 and 0 to the power of any value is 0.

<u>Example</u>

Write the equation for the following graph.



1. Locate the asymptote   Horizontal asymptote: $y = -2$

2. Locate the point one unit from the asymptote
   The point is -1 units away from the y – axis

3. Substitute the values of a and b into the equation

$$y = p^{x+1} - 2$$

4. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is $(0, -5)$

$$y = p^{x+2} - 2$$
$$-5 = p^{0+1} - 2$$
$$-3 = p^1$$

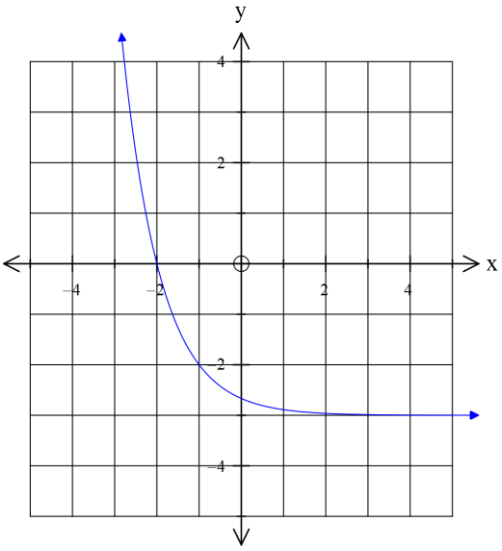
Substitute in (x, y)

Simplify $0 + 1$ and add 2 to both sides

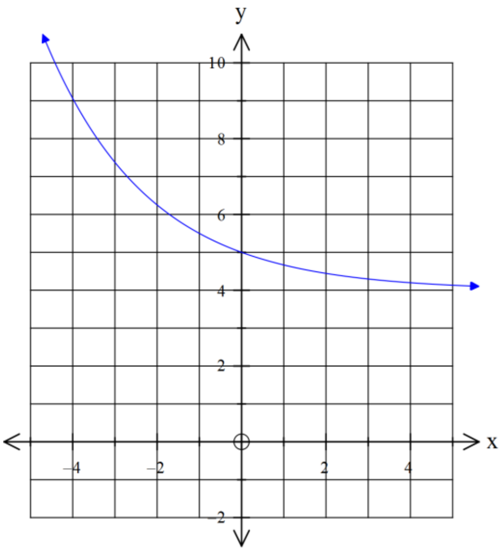
Equations of Exponential Functions

Write the equations of the following exponential functions

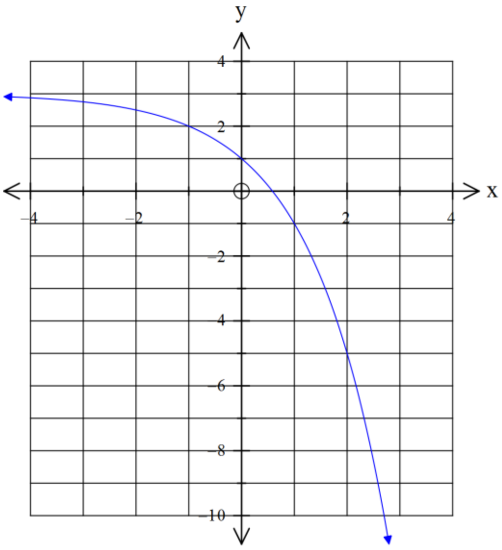
QUESTION ONE



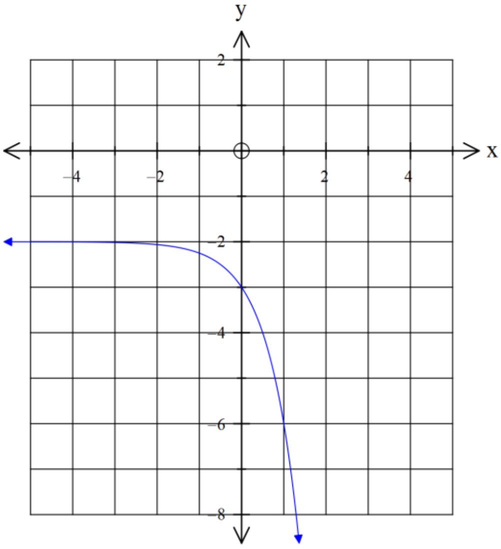
QUESTION TWO



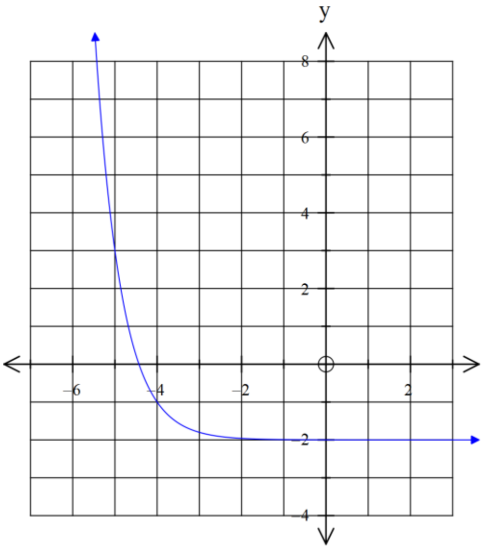
QUESTION THREE



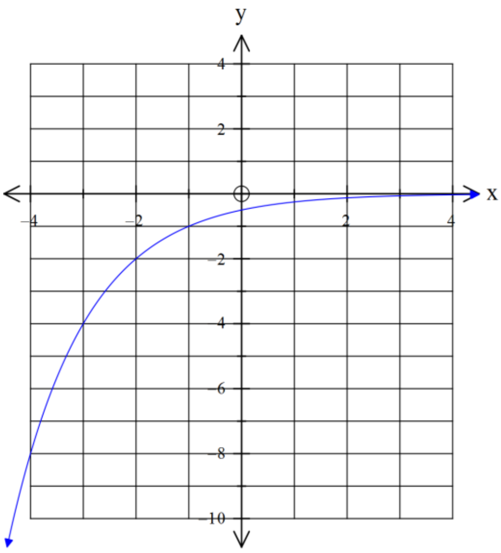
QUESTION FOUR



QUESTION FIVE



QUESTION SIX



Key Features of Exponential Functions

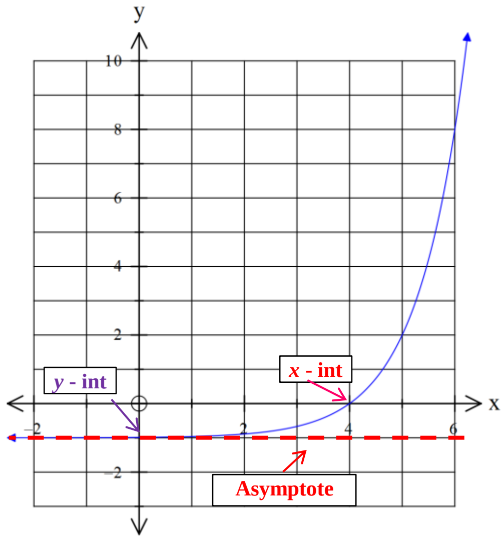
Example

State the key features for the exponential function $y = 3^{x-4} - 1$

Input the equation into your graphics calculator and use SHIFT F5 to locate key features

domain	no domain
x – intercept	(4, 0)
y – intercept	(0, −0.9877)
asymptote	$y = -1$

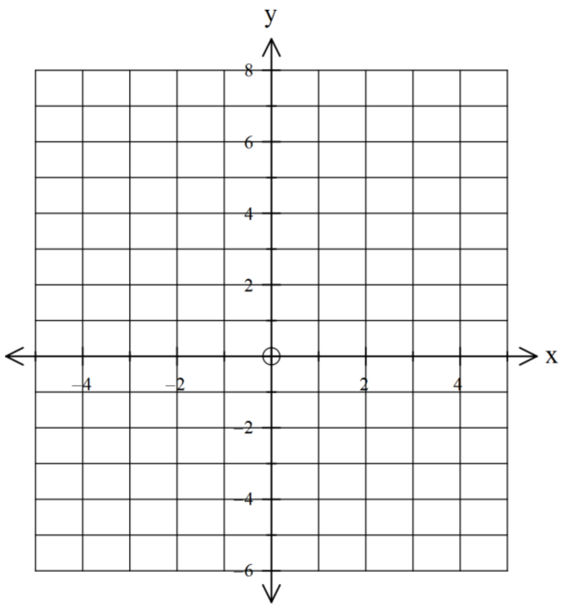
Note that the calculator cannot identify the asymptote; you need to be able to do this yourself.



Sketching Exponential Functions

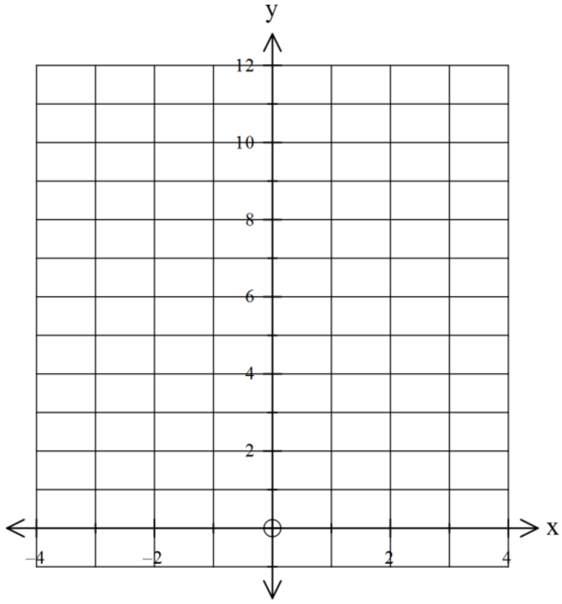
Graph each exponential function by finding the key features using your graphics calculators on the grids below.

QUESTION ONE



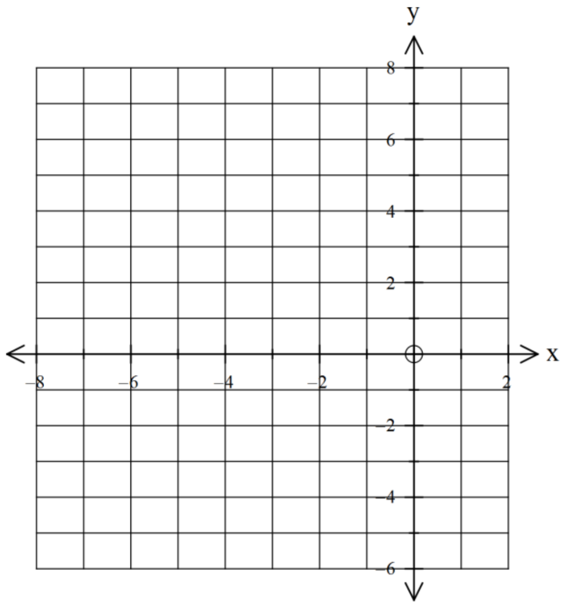
$y = 3^{x+1} - 5$	
domain	$x \leq 1$
x – intercept	
y – intercept	
horizontal asymptote	

QUESTION TWO



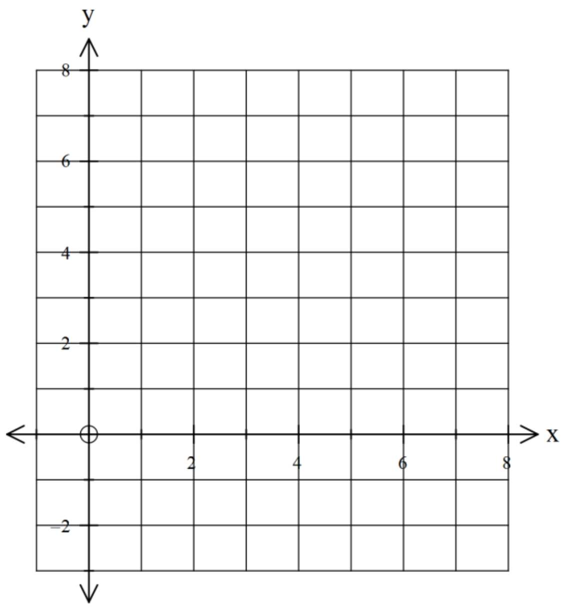
$y = 2^{x+1} + 3$	
domain	no domain
x – intercept	
y – intercept	
horizontal asymptote	

QUESTION THREE



$y = \left(\frac{3}{5}\right)^{x} - 5$	
domain	no domain
x – intercept	
y – intercept	
horizontal asymptote	

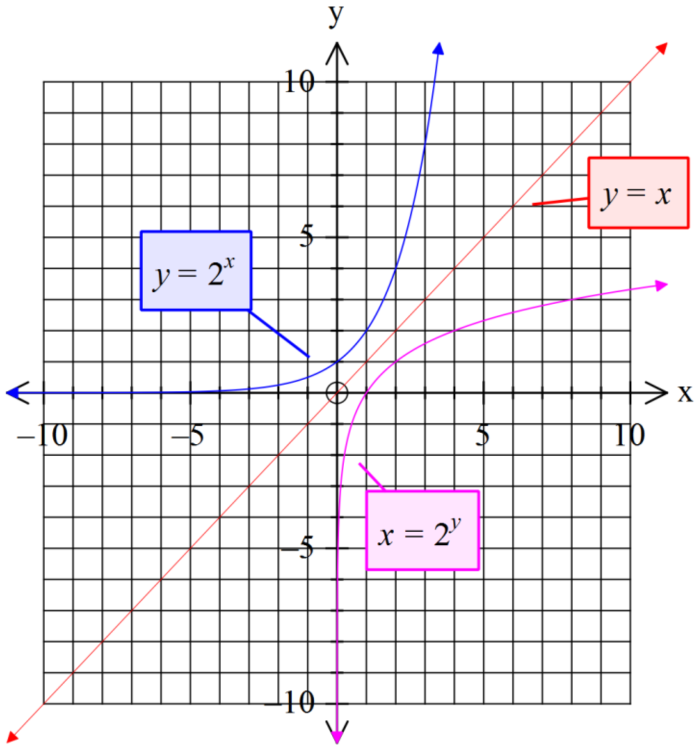
QUESTION FOUR



$y = -(2^{x-3}) + 5$	
domain	$x \leq 6$
x – intercept	
y – intercept	
horizontal asymptote	

Logarithms

If we reflect the exponential graph along the line $y = x$, we get a logarithm graph or log graph.



	Standard exponential	Standard logarithm
Equation	$y = p^x$	$x = p^y$
Asymptotes	Horizontal asymptote at $y = 0$	Vertical asymptote at $x = 0$
Point graph will always pass through	(1, 0)	(0, 1)

The **general equation** for a logarithm can be expressed as

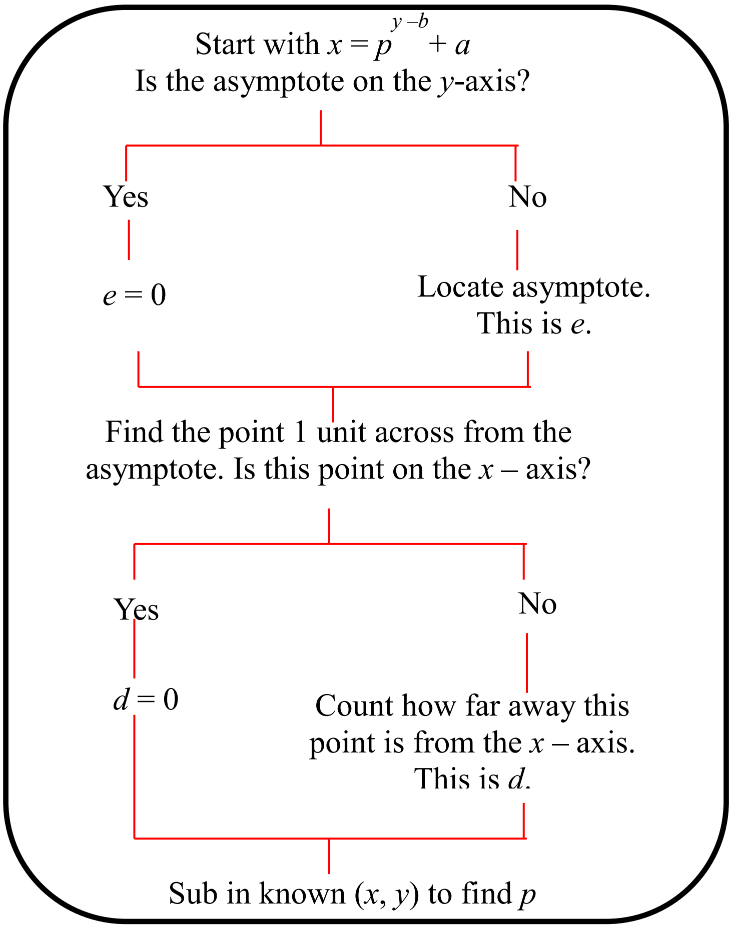
$$x = p^{y-b} + a$$

The distance the asymptote has moved **horizontally**

A constant that can make the graph steeper

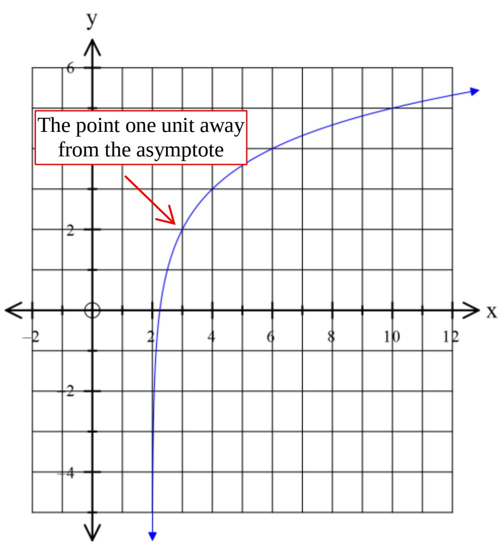
Select the point one unit across from the asymptote. The distance this point is from the x-axis is the **vertical movement**

The flowchart below can be used to assist in writing the equation of the logarithmic graph:



Example

Write the equation for the following graph.



1. Locate the asymptote   Vertical asymptote: $x = -2$

2. Locate the point one unit from the asymptote
   The point is $+ 2$ units away from the x – axis

3. Substitute the values of a and b into the equation

$$x = p^{y-2} - 2$$

4. To calculate k, locate another (x, y) coordinate on the graph and substitute into the equation.

   Selected point is $(4, 3)$

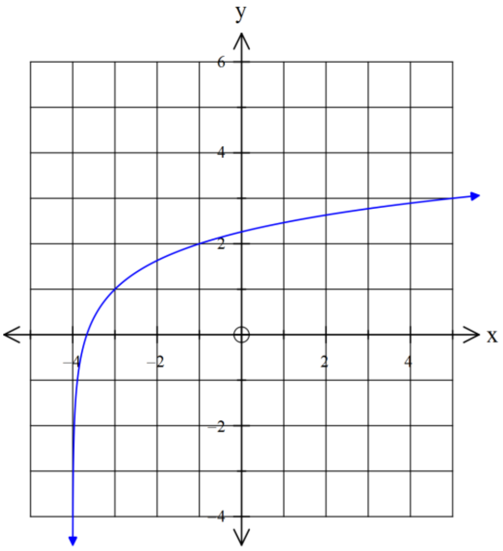
$$x = p^{y-2} - 2$$
Substitute in (x, y)
$$4 = p^{3-2} - 2$$
Simplify 3 – 2 and add 2 to both sides
$$2 = p^{1}$$

The equation of the logarithm is $x = 2^{y-2} - 2$

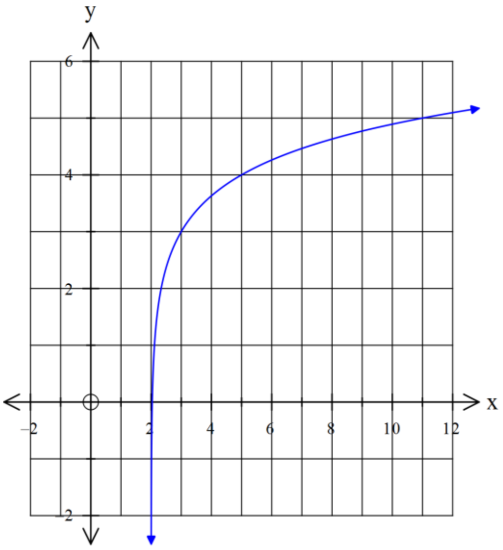
Equations of Logarithms

Write the equations of the following logarithmic functions

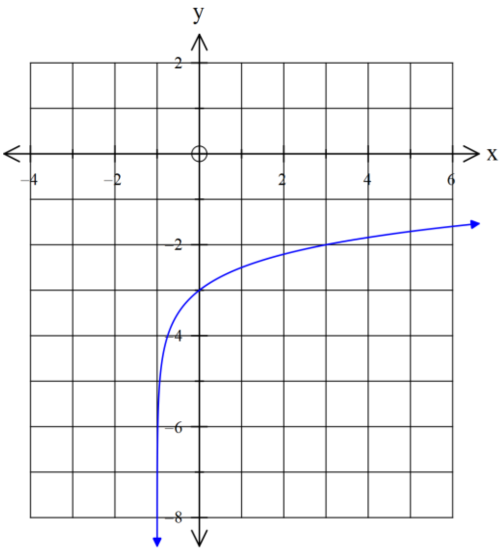
QUESTION ONE



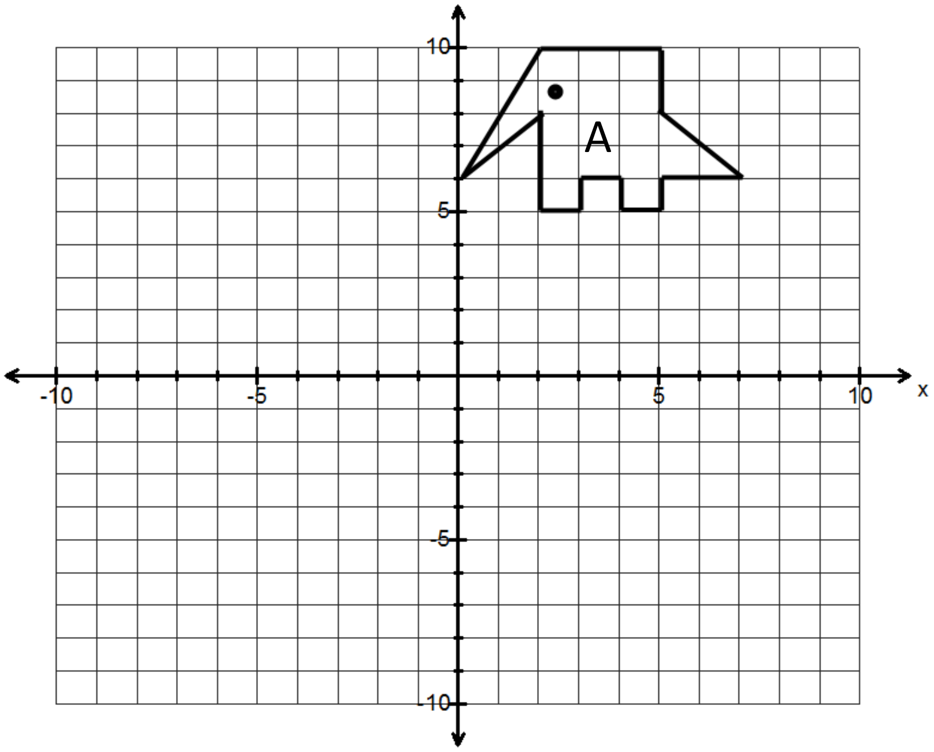
QUESTION TWO



QUESTION THREE



Transformations: Reflection, Translation and Enlargement



On the grid provided:

1. **Reflect** A in the *y* – axis. Draw and label this B.

2. **Translate** (move) B down 11 and to the right 3. Draw and label this C.

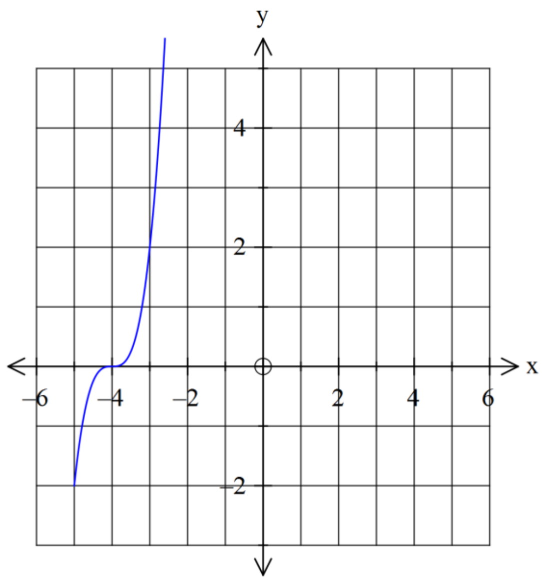
3. **Enlarge** C by a scale factor of 2 around (0, -5). In other words double the size of C using (0, -5) as a reference point. Draw and label D.

Reflection	Enlargement	Translation
All the points are reflected along a specified **mirror line.**	From a given point or from a point you have specified. If a **scale factor** is not provided, double its size.	Move all the points in a given direction. Specify the **vector of translation**

Transformation of Graphs (Merit)

Write the equations of the following graphs. Each graph will have instructions on how to transform it. Sketch the transformed graph on the same grid and compare the two graphs

QUESTION ONE



Reflect the cubic in the mirrorline $x = -3$

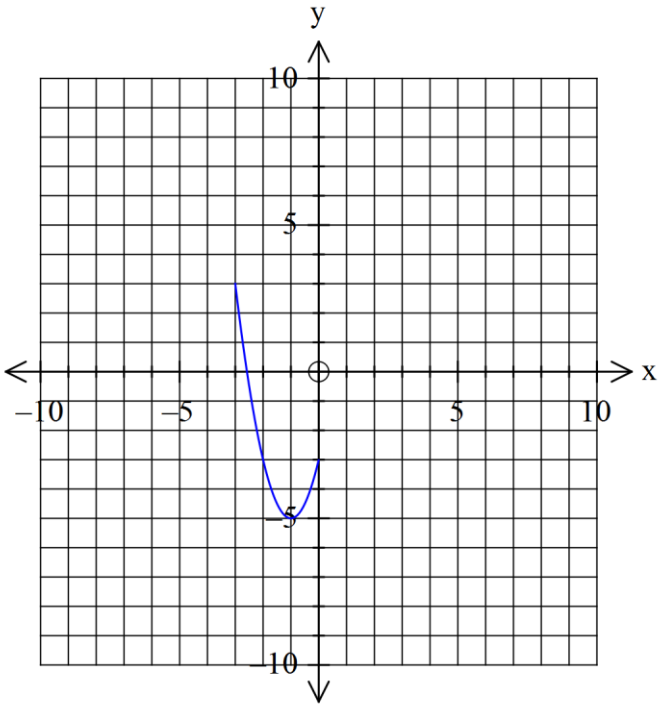
	Original Graph	Transformed Graph
equation		
domain		
point of inflection		

I notice that the reflected graph is different because

I notice that the equation is different because

I notice the domains are different because

QUESTION TWO



Enlarge the parabola by a scale factor of 2 around the co-ordinate $(-1, -5)$

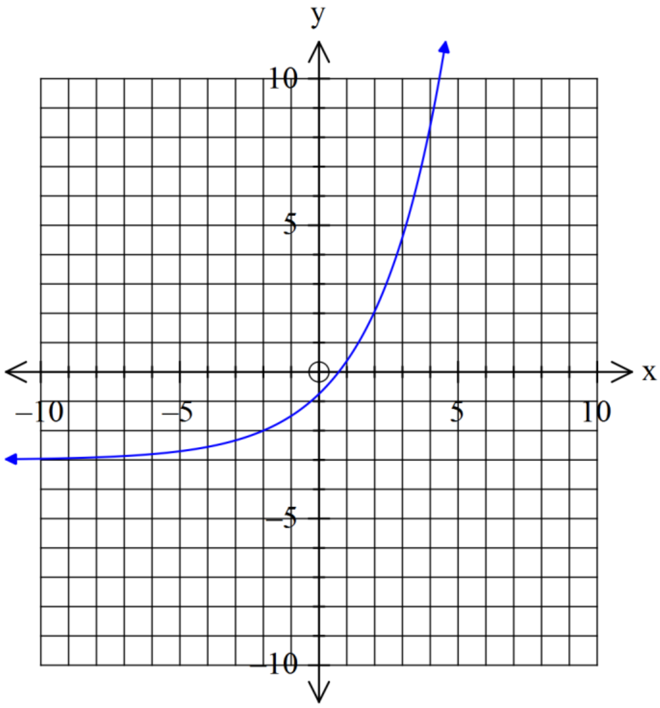
	Original Graph	Transformed Graph
equation		
domain		
vertex		
axis of symmetry		

I notice that the enlarged graph is different because

I notice that the equation is different because

I notice the domains are different because

QUESTION THREE



Translate the exponential 4 units right and 2 units down

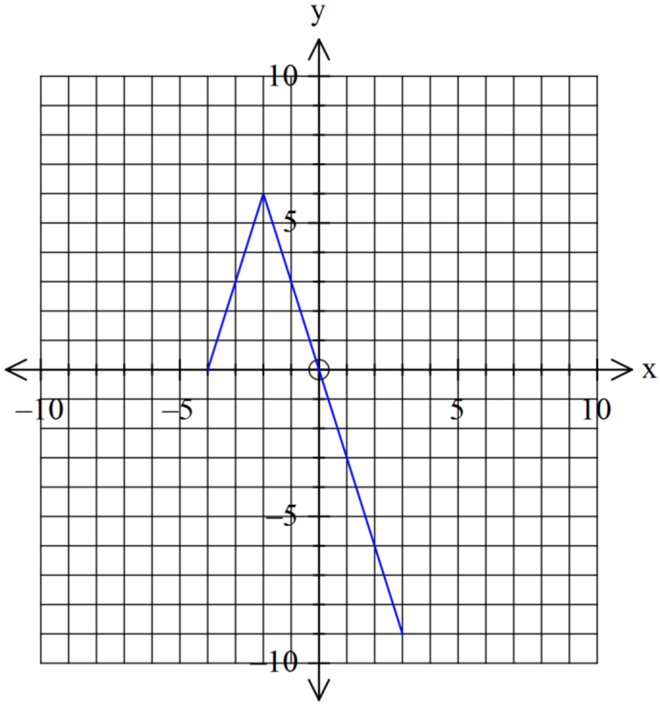
	Original Graph	Transformed Graph
equation		
domain		
asymptote		

I notice that the translated graph is different because

I notice that the equation is different because

I notice the domains are different because

QUESTION FOUR



Enlarge the absolute value function by a scale factor of 3 with the centre of enlargement being the vertex.

	Original Graph	Transformed Graph
equation		
domain		
vertex		
axis of symmetry		

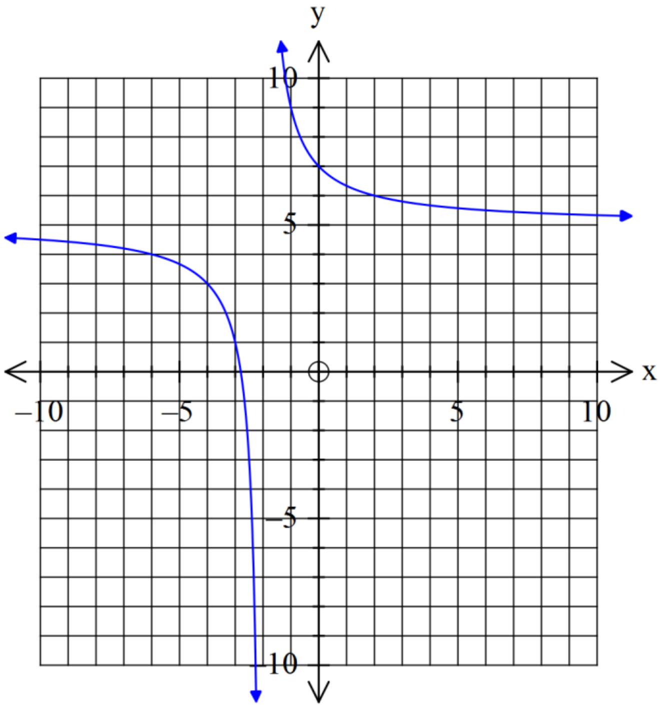
I notice that the enlarged graph is different because

I notice that the equation is different because

I notice the domains are different because

(Extra) Combined Transformation of Graphs

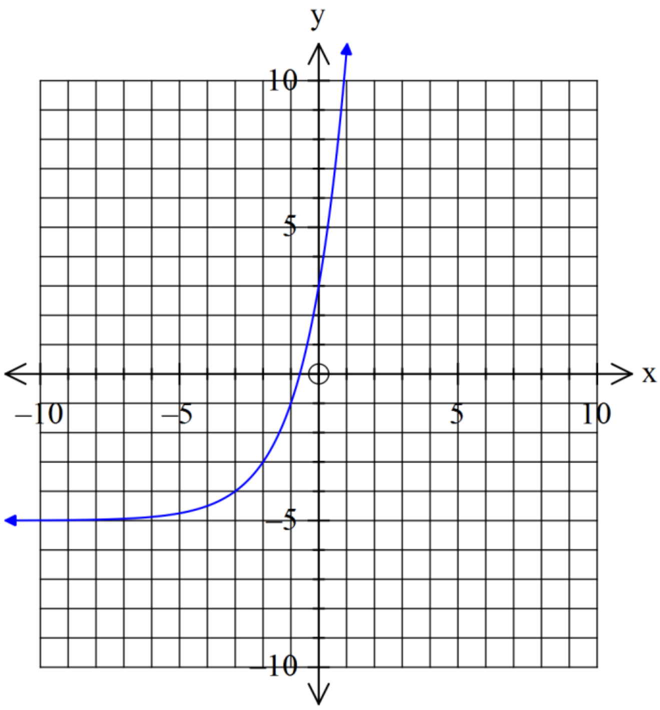
QUESTION ONE



Translate the hyperbola 5 units right and 2 units down. Reflect this graph along $y = 3$

	Equation
original	
new	

QUESTION TWO

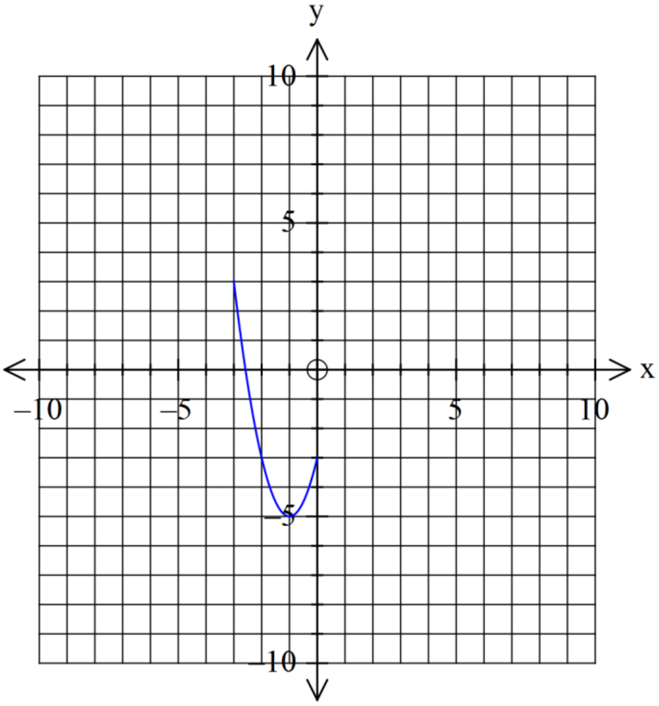


Reflect the exponential around the y – axis and shift it 3 units up and 2 units left

	Equation
original	
new	

General Solutions (Excellence)

Example



Enlarge the original parabola by different scale factors around the co-ordinate $(-1, -5)$

	equation	domain
Original parabola	$y = 2(x + 1)^2 - 5$	$-3 \leq x \leq 0$
Parabola (s.f. 2)	$y = (x + 1)^2 - 5$	$-5 \leq x \leq 1$
Parabola (s.f. 3)	$y = \frac{2}{3}(x + 1)^2 - 5$	$-7 \leq x \leq 2$
Parabola (s.f. 4)	$y = \frac{1}{2}(x + 1)^2 - 5$	$-9 \leq x \leq 3$

We can find the equation (a domain) of the original parabola with different scale factors by sketching it on the grid above.

Notice that a pattern forms for the equations when the original parabola is enlarged.

In this case, the only values that alter in the equation is the scale factor.

Pattern in scale factor:

Original	s.f. 2	s.f. 3	s.f. 4
$\frac{2}{1} = 2$	$\frac{2}{2} = 1$	$\frac{2}{3}$	$\frac{2}{4} = \frac{1}{2}$

We notice that in the pattern, the numerator stays the same, but the denominator increases by 1 each time the scale factor of the enlargement increases by 1.

This means that if the enlargement scale factor is n, the equation's scale factor is $\frac{2}{n}$.

The domains of the equation have also altered.

Pattern in the left hand domain: −3, −5, −7, −9.

This means that if the enlargement scale factor is n, the left hand domain is $-2n-1$

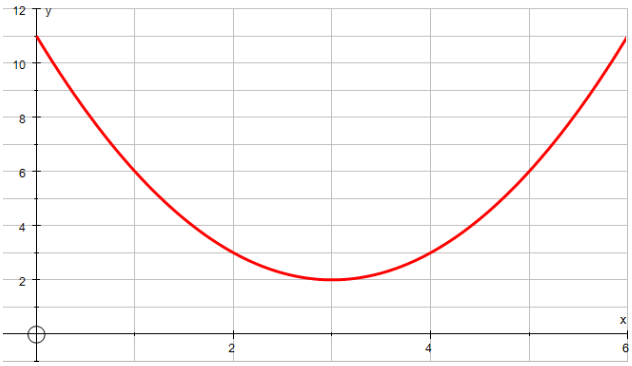
Pattern in the right hand domain: 0, 1, 2, 3.

This means that if the enlargement scale factor is n, the right hand domain is $n-1$

Therefore, when the enlargement scale factor is n on the graph $y = 2(x+1)^2 - 5$, the equation can be written as $y = \frac{2}{n}(x+1)^2 - 5$ with domains $-2n-1 \le x \le n-1$.

Finding the General Solution

1. Find the equation of this parabola and its domains



2. Enlarge the parabola by scale factor 2 with centre of enlargement (3,0)

   Find the new equation and domain

3. Repeat with scale factor 3, centre of enlargement (3, 0)

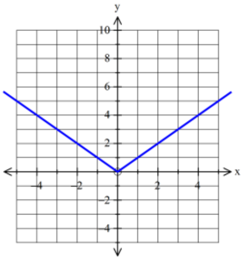
4. Repeat with scale factor 4, centre of enlargement (3,0)

What pattern can you see in the equations?

Use this pattern to determine the equation and domain for an enlargement of scale factor of 10, centre (3,0)

Summary of Graphs

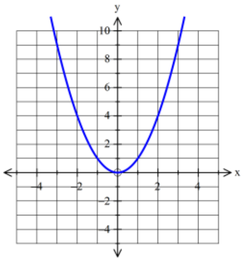
Absolute value



$y = k\,|x - a| + b$

- domain
- x – intercept(s)
- y - intercept
- vertex
- axis of symmetry

Parabola

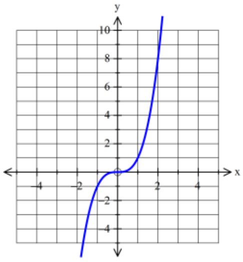


$y = k\,(x - a)^2 + b$

$y = k\,(x - c)\,(x - d)$

- domain
- x – intercept(s)
- y - intercept
- vertex
- axis of symmetry

Cubic

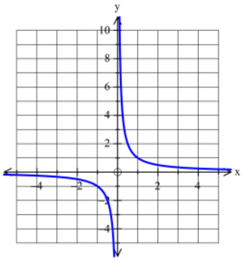


$y = k\,(x - a)^3 + b$

$y = k\,(x - c)\,(x - d)\,(x - e)$

- domain
- x – intercept(s)
- y - intercept
- point of inflection

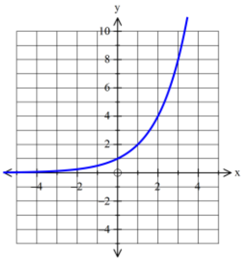
Hyperbola



$$y = \frac{k}{x - a} + b$$

- domain
- x – intercept(s)
- y - intercept
- vertical asymptote
- horizontal asymptote

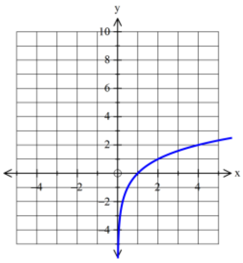
Exponential



$y = p^{x - a} + \mathrm{b}$

- domain
- x – intercept(s)
- y - intercept
- horizontal asymptote

Logarithm



$x = p^{y - b} + a$

- domain
- x – intercept(s)
- y - intercept
- vertical asymptote

***a* = horizontal shift, *b* = vertical shift and *k* = scale factor**

Answers

Equations of Absolute Value Graphs

p. 4 – 5

1. $y = \frac{2}{3}|x+4| - 7$
2. $y = -3|x| + 7$
3. $y = \frac{1}{3}|x-3| - 3$
4. $y = \frac{7}{6}|x+5| + 3$
5. $y = \frac{4}{5}|x-2| + 1$
6. $y = -2|x+5| + 6$

Sketching Absolute Value Graphs

p. 6 – 7

1. x-intercept (3, 0)
   y – intercept (0, 2)
   vertex (3, 0)
   line of symmetry $x = 3$
2. x-intercept (−3, 0) (1, 0)
   y – intercept (0, −2)
   vertex (−1, −4)
   line of symmetry $x = -1$
3. x-intercept (−8, 0) (0, 0)
   y – intercept (0, 0)
   vertex (−4, 6)
   line of symmetry $x = -4$
4. x-intercept $\left(-\frac{5}{3}, 0\right)\left(\frac{5}{3}, 0\right)$
   y – intercept (0, −5)
   vertex (0, −5)
   line of symmetry $x = 0$

Equations of Parabolas

p. 10 – 11

1. $y = 2(x-5)^2 - 4$
2. $y = \frac{2}{7}(x+3)(x-5)$
3. $y = -3(x+2)(x-1)$
4. $y = \frac{5}{8}(x-2)^2 - 8$
5. $y = -\frac{2}{3}(x-4)^2 + 5$
6. $y = \frac{2}{27}x(x+3)$

Sketching Parabolas

p.12 – 14

1. x-intercept (3, 0)
   y – intercept (0, 27)
   vertex (3, 0)
   line of symmetry $x = 3$
2. x-intercept (−1, 0) (3, 0)
   y – intercept (0, 6)
   vertex (1, 8)
   line of symmetry $x = 1$
3. x-intercept (−5, 0) (7, 0)
   y – intercept (0, −10)
   vertex (1, −10.29)
   line of symmetry $x = 1$
4. x-intercept (−3.66, 0) (7.66, 0)
   y – intercept (0, 7)
   vertex (2, 8)
   line of symmetry $x = 2$
5. x-intercept (−5, 0) (1, 0)
   y – intercept (0, 2)
   vertex (−2, 3.6)
   line of symmetry $x = -2$
6. x-intercept none
   y – intercept (0, 52)
   vertex (−5, 2)
   line of symmetry $x = -5$
7. x-intercept (−1, 0) (2, 0)
   y – intercept (0, −6)
   vertex (0.5, −6.75)
   line of symmetry $x = 0.5$

Equations of Cubics

p.17 – 18

1. $y = 3(x+2)^3 - 6$
2. $y = 4(x-5)^3 + 2$
3. $y = -\frac{1}{4}(x+1)^3 - 2$
4. $y = \frac{2}{5}(x+5)(x+1)(x-2)$
5. $y = -\frac{1}{2}(x+4)(x+3)(x-2)$
6. $y = 2(x+1)(x-1)(x-2)$

Sketching Cubics
p. 20

1. x-intercept (−3.88, 0)
   y – intercept (0, 11)
   point of inflection (−2, 5)
2. x-intercept (4.793, 0)
   y – intercept (0, 129)
   point of inflection (4, 1)
3. x-intercept (−1, 0) (1, 0) (2, 0)
   y – intercept (0, 2.4)
   maximum (−0.22, 2.54)
   minimum (1.55, −0.76)

Equations of Higher Powered Polynomials
p.22 – 23

1. $y = -\frac{1}{4}(x+3)(x+2)(x-1)^2$
2. $y = \frac{1}{9}(x+3)(x+2)(x+1)(x-3)$
3. $y = \frac{3}{50}x(x+3)^2(x-3)^2$
4. $y = \frac{1}{10}x(x+4)(x+1)(x-1)(x-3)$

Equations of Rectangular Hyperbolae
p.25 – 26

1. $y = \frac{2}{x-3} + 2$
2. $y = \frac{3}{x+2} - 3$
3. $y = \frac{4}{x+1}$
4. $y = \frac{3}{x-2} + 1$

Sketching Rectangular Hyperbolae
p.27 – 28

1. x-intercept (0, 0)
   y – intercept (0, 0)
   vertical asymptote $x = 1$
   horizontal asymptote $y = 2$
2. x-intercept (0.5, 0)
   y – intercept (0, 0.25)
   vertical asymptote $x = 4$
   horizontal asymptote $y = 2$
3. x-intercept none
   y – intercept (0, 3)
   vertical asymptote $x = 1$
   horizontal asymptote $y = 0$
4. x-intercept $\left(\frac{1}{3}, 0\right)$
   y – intercept none
   vertical asymptote $x = 0$
   horizontal asymptote $y = 3$

Equations of Exponential Functions
p.30 – 31

1. $y = 2^{x-5} - 2$
2. $y = 1.49^{x+3}$
3. $y = 3^{x+2} + 3$
4. $y = 2.45^{x-1} - 3$

Equations of Exponential Functions
p.33 – 34

1. $y = \frac{1}{3}^{x+1} - 3$
2. $y = \frac{2}{3}^{x} + 4$
3. $y = -2^{x+1} + 3$
4. $y = -4^{x} - 2$
5. $y = \frac{1}{5}^{x+4} - 2$
6. $y = -\frac{1}{2}^{x+1}$

Sketching Exponential Functions
p.35 – 36

1. x-intercept (0.465, 0)
   y – intercept (0, −2)
   horizontal asymptote $y = -5$
2. x-intercept none
   y – intercept (0, 5)
   horizontal asymptote $y = 3$
3. x-intercept (−3.15, 0)
   y – intercept (0, −4)
   horizontal asymptote $y = -5$
4. x-intercept (5.32, 0)
   y – intercept (0, 4.875)
   horizontal asymptote $y = 5$

Equations of Logarithms
p.39

1. $x = 3^{y-1} - 4$
2. $x = 3^{y-3} + 2$
3. $x = 2^{y+3} - 1$

Transformations of Graphs

p.41 – 44

1.

	Original Graph	Transformed Graph
equation	$y = 2(x + 4)^3$	$y = -2\ (x + 2)^2$
domain	$x \geq -5$	$x \leq -1$
point of inflection	$(-4, 0)$	$(-2, 0)$

2.

	Original Graph	Transformed Graph
equation	$y = 2(x + 1)^2 - 5$	$y = (x + 1)^2 - 5$
domain	$-3 \leq x \leq 0$	$-5 \leq x \leq 1$
vertex	$(-1, -5)$	$(-1, -5)$
axis of symmetry	$x = -1$	$x = -1$

3.

	Original Graph	Transformed Graph
equation	$y = 1.49^{x+2} - 3$	$y = 1.49^{x-2} - 5$
domain	none	None
asymptote	$y = -3$	$y = -5$

4.

	Original Graph	Transformed Graph
equation	$y = -3\lvert x + 2\rvert + 6$	$y = -3\lvert x + 2\rvert + 6$
domain	$-4 \leq x \leq 3$	$-8 \leq x \leq 13$
vertex	$(-2,6)$	$(-2,6)$
axis of symmetry	$x = -2$	$x = -2$

Extra: Combined Transformations of Graphs

p.45

1.

	Equation
original	$y = \dfrac{4}{x+2} + 5$
new	$y = -\dfrac{4}{x+3} + 3$

2.

	Equation
original	$y = 2^{x+3} - 5$
new	$y = \dfrac{1}{2}^{x+1} - 2$

General Solutions

p.47

$y = (x-3)^2 + 2$	$0 \leq x \leq 6$	$y = \frac{1}{4}(x-3)^2 + 2$	$-9 \leq x \leq 15$
$y = \frac{1}{2}(x-3)^2 + 2$	$-3 \leq x \leq 9$	$y = \frac{1}{n}(x-3)^2 + 2$	$-(3n + 3) \leq x \leq 3n + 3$
$y = \frac{1}{3}(x-3)^2 + 2$	$-6 \leq x \leq 12$	$y = \frac{1}{10}(x-3)^2 + 2$	$-27 \leq x \leq 33$